Almost every aspect of life presents us with decision problems, ranging from the simple question of whether to have pizza or ice cream, or where to aim a penalty kick, to more complex decisions like how a company should compete with others and how governments should negotiate treaties. Game theory is a technique that can be used to analyse strategic problems in diverse settings; its application is not limited to a single discipline such as economics or business studies. A Guide to Game Theory reflects this interdisciplinary potential to provide an introductory overview of the subject.

Put off by a fear of maths? No need to be, as this book explains many of the important concepts and techniques without using mathematical language or methods. This will enable those who are alienated by maths to work with and understand many game theoretic techniques.

**KEY FEATURES**

- Key concepts and techniques are introduced in early chapters, such as the prisoners’ dilemma and Nash equilibrium. Analysis is later built up in a step-by-step way in order to incorporate more interesting features of the world we live in.
- Using a wide range of examples and applications the book covers decision problems confronted by firms, employers, unions, footballers, partygoers, politicians, governments, non-governmental organisations and communities.
- Exercises and activities are embedded in the text of the chapters and additional problems are included at the end of each chapter to test understanding.
- Realism is introduced into the analysis in a sequential way, enabling you to build on your knowledge and understanding and appreciate the potential uses of the theory.

Suitable for those with no prior knowledge of game theory, studying courses related to strategic thinking. Such courses may be a part of a degree programme in business, economics, social or natural sciences.

**FIONA CARMICHAEL** is Senior Lecturer in Economics at the University of Salford. She has a wealth of experience in helping students tackle this potentially daunting yet fascinating subject, as recognised by an LTSN award for ‘Outstanding Teaching’ on her innovative course in game theory.
A Guide to Game Theory
We work with leading authors to develop the strongest educational materials in game theory, bringing cutting-edge thinking and best learning practice to a global market.

Under a range of well-known imprints, including Financial Times Prentice Hall, we craft high quality print and electronic publications which help readers to understand and apply their content, whether studying or at work.

To find out more about the complete range of our publishing, please visit us on the World Wide Web at: www.pearsoned.co.uk
To Jessie and Rosie
CONTENTS

Preface xi
Acknowledgements xiv
Publisher's acknowledgements xv

CHAPTER 1  Game theory toolbox  1

Introduction  2
1.1 The idea of game theory  3
1.2 Describing strategic games  5
1.3 Simultaneous-move games  7
1.4 Sequential-move or dynamic games  13
1.5 Repetition  16
1.6 Cooperative and non-cooperative games  16
1.7 N-player games  17
1.8 Information  17
Summary  18
Answers to exercises  19
Problems  20
Questions for discussion  20
Notes  20

CHAPTER 2  Moving together  21

Introduction  22
2.1 Dominant-strategy equilibrium  22
2.2 Iterated-dominance equilibrium  29
2.3 Nash equilibrium  36
2.4 Some classic games  43
Summary  50
Answers to exercises  51
Problems  53
Questions for discussion  54
CHAPTER 3  Prisoners’ dilemma  57

Introduction  58
3.1 Original prisoners’ dilemma game  58
3.2 Generalised prisoners’ dilemma  60
3.3 Prisoners’ dilemma and oligopoly collusion  62
3.4 International trade  64
3.5 Prisoners’ dilemma and public goods  66
3.6 Prisoners’ dilemma and open-access resources  68
3.7 Macroeconomics  70
3.8 Resolving the prisoners’ dilemma  71
Summary  72
Answers to exercises  73
Problems  74
Questions for discussion  75
Answers to problems  75
Notes  76

CHAPTER 4  Taking turns  79

Introduction  80
4.1 Foreign direct investment game  81
4.2 Nice–not so nice game  89
4.3 Trespass  93
4.4 Entry deterrence  96
4.5 Centipede games  100
Summary  103
Answers to exercises  104
Problems  105
Questions for discussion  106
Answers to problems  106
Notes  107

CHAPTER 5  Hidden moves and risky choices  109

Introduction  110
5.1 Hidden moves  110
5.2 Risk and probabilities  113
5.3 Limitations of expected utility theory  125
Summary  135
Answers to exercises  136
Problems  137
Questions for discussion  137
### CHAPTER 6  
**Mixing and evolving**

**Introduction**

6.1 Nash equilibrium in mixed strategies

6.2 Evolutionary games

*Summary*

*Answers to exercises*

*Problems*

*Questions for discussion*

*Answers to problems*

*Notes*

---

### CHAPTER 7  
**Mystery players**

**Introduction**

7.1 Friends or enemies again

7.2 Entry deterrence with incomplete information

7.3 Entry deterrence with signalling

7.4 Numerical example of entry deterrence with signalling

7.5 The beer and quiche signalling game

7.6 Asymmetric information for both players in the battle of the sexes

*Summary*

*Answers to exercises*

*Problems*

*Questions for discussion*

*Answers to problems*

*Notes*

---

### CHAPTER 8  
**Playing again and again . . .**

**Introduction**

8.1 Finite repetition

8.2 Infinite and indefinite repetition

8.3 Asymmetric information in the finitely repeated prisoners’ dilemma

8.4 Resolving the chain store paradox

8.5 Experimental evidence

*Summary*

*Answers to exercises*

*Problem*

*Questions for discussion*

*Answer to problem*

*Notes*
CHAPTER 9  Bargaining and negotiation  235

Introduction  236
9.1 Cooperative and non-cooperative bargaining theory  236
9.2 Bargaining problem  237
9.3 Cooperative bargaining theory  241
9.4 Non-cooperative, strategic bargaining with alternating offers  249
9.5 Experimental evidence  263
Summary  265
Answers to exercises  266
Problems  267
Questions for discussion  267
Answers to problems  268
Notes  268

Bibliography  271
Index  279
This book gives an introductory overview of game theory. It has been written for people who have little or no prior knowledge of the theory and want to learn a lot without getting bogged down in either thousands of examples or mathematical quicksand. Game theory is a technique that can be used to analyse strategic problems in diverse settings. Its application is not limited to a single discipline such as economics or business studies and this book reflects this interdisciplinary potential. A wide range of examples and applications are used including decision problems confronted by firms, employers, unions, footballers, partygoers, politicians, governments, non-governmental organisations and communities. Students on different social and natural sciences programmes where game theory is part of the curriculum should therefore find this book useful. It will be particularly helpful for students who sometimes feel daunted by mathematical language and expositions. I have written it with them in mind and have kept the maths to a minimum to prevent it from becoming overbearing.

Mathematical language can act as a barrier that stops theories like game theory, that have their origins in mathematics, from being applied elsewhere. This book aims to break down these barriers and the exposition relies heavily on a logical approach aided by tables and diagrams. Often this is all that is needed to convey the essential aspects of the scenario under investigation. However, this won’t always be the case and sometimes, in order to get closer to the real world, it is helpful to use mathematical language in order to give precision to what might otherwise be very long and possibly rambling explanations.

In the first four chapters of this book you will learn about many of the important ideas in game theory: concepts like zero-sum games, the prisoners’ dilemma, Nash equilibrium, credible threats and more. In the subsequent chapters the analysis is built up in a step-by-step way in order to incorporate more of the interesting features of the world we live in, such as risk, information asymmetries, signals, long-term relationships, learning and negotiation. Naturally, the insights generated by the theory are likely to be more useful the
greater the degree of reality incorporated into the analysis. The trade-off is that
the more closely the analysis mirrors the real world the more intricate it
becomes. To help you thread your way through these intricacies a small
number of examples are followed through and analysed in detail. An alterna-
tive approach might be to build on the material in the earlier chapters by
applying it in some specific but relatively narrowly-defined circumstances. This
alternative would bypass many of the potential uses of game theory and, I
think, do you and the theory a disservice.

As you read through the chapters in this book you will find that there are
plenty of opportunities for you to put into practice the game theory you learn
by working through puzzles, or more formally in the language of the class-
room, exercises and problems. The exercises are embedded in the text of the
chapters and there are additional problems and discussion questions at the end
of the chapters. Working through problems is a really good way of testing your
understanding and you may find that learning game theory is a bit like learn-
ing to swim or ride a bike in that it is something that you can only really
understand by doing.

The plan of this book is as follows. In Chapter 1, some of the basic ideas and
concepts underlying game theory are outlined and some examples are given of
the kinds of scenario where game theory can be applied usefully. The objectives
of using game theory in these circumstances are also discussed. In Chapter 2
simultaneous- or hidden-move games are analysed and the dominant strategy
and Nash equilibrium concepts are defined. Some limitations of these solution
concepts are also discussed.

The subject of Chapter 3 is the prisoners’ dilemma, a famous hidden-move
game. In Chapter 3 you will see how the prisoners’ dilemma can be generalised
and set in a variety of contexts. You will see that some important questions are
raised by the prisoners’ dilemma in relation to decision theory in general and
ideas of rationality in particular. Examples of prisoners’ dilemmas in the social,
business and political spheres of life are explored. Some related policy ques-
tions in connection with public and open access goods and the free rider effect
are analysed in depth using examples.

Dynamic games are analysed in Chapter 4 and you will learn how sequential
decision making can be modelled using game theory and extensive forms.
Examples are used to demonstrate why the idea of a Nash equilibrium on its
own may not be enough to solve dynamic games. Backward induction is used
to show that only a refinement of the Nash equilibrium concept, called a sub-
game perfect Nash equilibrium, rules out non-credible threats. Games
involving threats to prosecute trespassers and fight entry are used to explore
the idea of commitment. The centipede game is also analysed and some ques-
tions are raised about the scope of the backward induction method.

All the games analysed in Chapters 1 to 4 involve an element of risk for the
participants as they won’t usually know what the other participants are going to
do. This kind of information problem is central to the analysis of games. In
Chapters 5 to 7 the analysis is extended to allow for even more of the risks and
uncertainties that abound in the world we live in. In Chapter 5 you will see how hidden and chance moves are incorporated into game theory and decision theory more generally. Expected values and expected utilities are compared. Attitudes to risk are discussed and examples are used to explain the significance of risk aversion and risk neutrality. The experimental evidence relating to expected utility theory is considered in detail and the implications of that evidence for the predictive powers and normative claims of the theory are discussed.

In Chapter 6 the Nash equilibrium concept is extended to incorporate randomising or mixed strategies. Randomisation won’t always appeal to individual players but can make sense in terms of a group or population of players. This possibility is explored in the context of evolutionary game theory. Some familiar examples such as chicken, coordination with assurance in the stag hunt game and the prisoners’ dilemma are used to examine some of the key insights of evolutionary game theory. The concept of an evolutionary stable equilibrium is explained and used to explore ideas relating to natural selection and the evolution of social conventions.

In Chapter 7 the analysis of the previous chapters is extended by allowing for asymmetric information in one-shot games. Examples, some from previous chapters (such as the entry deterrence game and the battle of the sexes) and some that are new like the beer and quiche game, are developed to explain how incomplete information about players’ identities changes the outcome of games. Bayes’ rule and the idea of a Bayesian equilibrium are introduced. The role of signalling in dynamic games with asymmetric information is explored.

In Chapter 8 more realism is incorporated by allowing for the possibility that people play some games more than once. Backward induction is used to solve the finitely repeated prisoners’ dilemma and the entry deterrence game. A paradox of backward induction is resolved by allowing for uncertainty about either the timing of the last repetition of the game, players’ pay-offs or their state of mind. The prisoners’ dilemma and the entry deterrence game are developed to allow for these kinds of uncertainties. In Chapter 9, the methodology used to analyse dynamic games in Chapter 4 is applied to strategic bargaining problems. In addition you will see some cooperative game theory. Nash’s bargaining solution and the alternating-offers model are both outlined and bargaining solutions are derived for a number of examples. The related experimental evidence is also considered.

I hope that you enjoy working through the game theory in this book and that you find the games in it both interesting and challenging.

Lecturers can additionally download an Instructor’s Manual and PowerPoint slides from http://www.booksites.net/carmichael.
This book would not have been possible without the help of a number of people. They include Gerry Tanner who was constantly available for all kinds of advice. I also need to thank Dominic Tanner for his artwork. Claire Hulme pre-read most of the chapters. Sue Charles and Judith Mehta read the chapters that Claire didn’t. I am grateful to all three of them for their comments. I also need to thank the reviewers who, at the outset of this project, made many useful suggestions. All the students on the Strategy and Risk module at the University of Salford who test drove the chapters deserve credit. A number of them, Carol, David, John and Mario in particular, noticed mistakes that I had missed. Unfortunately, the mistakes that remain are down to me. Lastly I need to thank two non-humans, Jessie and Rosie, who make the occasional appearance.
We would like to express our gratitude to the following academics, as well as additional anonymous reviewers, who provided invaluable feedback on this book in the early stages of its development.

Mark Broom, University of Sussex
Jonathan Cave, University of Warwick
Roger Hartley, Keele University
GAME THEORY TOOLBOX

Concepts and techniques

- Strategic interdependence
- Players
- Strategies
- Pay-offs
- Utility
- Equilibrium
- Simultaneous-move games, static games
- Strategic form, pay-off matrix
- Sequential-move games, dynamic games
- Extensive form, game tree
- Repeated games
- Constant-sum and zero-sum games
- Cooperative games.

After working through this chapter you will be able to:
- Describe a strategic situation as a game
- Explain the difference between simultaneous moves and sequential moves in games
This chapter sets out a framework for understanding and applying game theory. It provides you with the tools that will enable you to use game theory to analyse a range of different problems. The general approach of game theory is outlined in the first part of the chapter; what it is and how and when it can be used. You will also see some examples of situations that could be usefully analysed as games. Some of the everyday language used by game theorists is explained and the type of outcome predicted by game theory is characterised. Two main categories of games are simultaneous-move games and sequential-move or dynamic games. These are both described in this chapter. You will see how pay-off matrices are used to capture the salient features of simultaneous-move games and how extensive forms or game trees are used to illustrate dynamic games. Games can be played only once or repeated, they can be co-operative or non-co-operative. Sometimes the participants in a game have shared interests and sometimes they don’t. These distinctions are all explained. In some games the participants will have the same information and in others they won’t. The amount of information in a game can affect its outcome and this possibility is discussed in the last section of this chapter. In the subsequent chapters of this book, the terminology that you are introduced to in this chapter and the different approaches that are outlined, will be developed so that you use game theory to interpret, explain and make predictions about the likely outcomes of decision problems that can be analysed as games.
The first important text in game theory was *Theory of Games and Economic Behaviour* by the mathematicians John von Neumann and Oskar Morgenstern published in 1944.¹ Game theory has evolved considerably since the publication of von Neuman and Morgenstern’s book and its reach has extended far beyond the confines of mathematics. This is due in a large part to contributions in the 1950s from John Nash (1950, 1951). However, it was in the 1970s that game theory as a way of analysing strategic situations began to be applied in all sorts of diverse areas including economics, politics, international relations, business and biology. A number of important publications precipitated this breakthrough, however, and Thomas Schelling’s book *The Strategy of Conflict* (1960) still stands out from a social science perspective.

Hutton (1996: 249) describes game theory as ‘an intellectual framework for examining what various parties to a decision should do given their possession of inadequate information and different objectives’. This definition describes what game theory can be used for rather than what it is. It also implicitly characterises the distinctive features of a situation that make it amenable to analysis using game theory. These features are that the actions of the parties concerned impact on each other but exactly how this might happen is unknown. Interdependence and information are therefore critical aspects of the definition of game theory.

Game theory is a technique used to analyse situations where for two or more individuals (or institutions) the outcome of an action by one of them depends not only on the particular action taken by that individual but also on the actions taken by the other (or others). In these circumstances the plans or strategies of the individuals concerned will be dependent on expectations about what the others are doing. Thus individuals in these kinds of situations are not making decisions in isolation, instead their decision making is interdependently related. This is called strategic interdependence and such situations are commonly known as games of strategy, or simply games, while the participants in such games are referred to as players. In strategic games the actions of one individual or group impact on others and, crucially, the individuals involved are aware of this.

Because players in a game are conscious that the outcomes of their actions are affected by and affect others they need to take into account the possible actions of these other individuals when they themselves make decisions. However, when individuals have limited information about other individuals’ planned actions (their strategies), they have to make conjectures about what they think they will do. These kinds of thought processes constitute strategic thinking and when this kind of thinking is involved game theory can help us to understand what is going on and make predictions about likely outcomes.²
Strategic thinking characterises many human interactions. Here are some examples:

(a) Two firms with large market shares in a particular industry making decisions with respect to price and output.

(b) Leaders of two countries contemplating a war with each other.

(c) The decision by a firm to enter a new market where there is a risk that the existing or incumbent firms will try to fight entry.

(d) Economic policy makers in a country contemplating whether to impose a tariff on imports.

(e) Leaders of two opposing factions in a civil war who are attempting to negotiate a peace treaty.

(f) Players taking/facing a penalty in association football.

(g) A tennis player deciding where to place a serve.

(h) Managers involved in the sale and purchase of players on the transfer market in association football.

(i) A criminal deciding whether to confess or not to a crime that he has committed with an accomplice who is also being questioned by the police.

(j) The decision by a team captain to declare in cricket.

(k) Family members arguing over the division of work within the household.

Games and who plays them

- **Strategic game**: a scenario or situation where for two or more individuals their choice of action or behaviour has an impact on the other (or others).

- **Strategic interdependence**: individuals’ decisions, their choices about actions, impact on each other and therefore their decision making is interdependently related.

- **Player**: a participant in a strategic game.

- **Strategy**: a player’s plan of action for the game.

In all of the above situations the participants or players are involved in a strategic game. The outcome of their planned actions depends on the actions of others players and therefore their plans may be thwarted in that they do not achieve their desired outcome. For example, in scenario (a) the players are firms with large market shares. Markets where a small number of large firms control a
large share of the market are called oligopolies. An example of an oligopoly is the automobile industry which is dominated by a small number of large multinational companies all of whom are household names (the top five in terms of sales are General Motors, Ford, Daimler Chrysler, Toyota and Volkswagen). Because the firms in an oligopoly are large relative to the size of the industry as a whole, the actions of the firms are independent. For instance, if one firm lowers its price the others are likely to lose custom to the price cutter, or if one firm raises its output by any significant amount the market price will probably fall. In both instances, the profits of the other firms will be lower because of the action of the first firm.

Exercise 1.1

In examples (b) to (k) above can you identify the players and explain why and how their actions are interdependent?

There are no wrong or necessarily right answers to Exercise 1.1 but just by thinking about examples like these you will be thinking about strategic situations. This means you will already be starting to think strategically.

Strategic thinking involves thinking about your interactions with others who are doing similar thinking at the same time and about the same situation. Making plans in a strategic situation requires thinking carefully before you act, taking into account what you think the people you are interacting with are also thinking about and planning. Because this kind of thinking is complex we need some sharp analytical tools in order to explain behaviour and predict outcomes in strategic situations – this is what game theory is for.

1.2 Describing strategic games

In order to be able to apply game theory a first step is to define the boundaries of the strategic game under consideration. Games are defined in terms of their rules. The rules of a game incorporate information about the players’ identity and their knowledge of the game, their possible moves or actions and their pay-offs. The rules of a game describe in detail how one player’s behaviour impacts on other players’ pay-offs. A player can be an individual, a couple, a family, a firm, a pressure group, the government, an intelligent animal – in fact any kind of thinking entity that is generally assumed to act rationally and is involved in a strategic game with one or more other players.

Players’ pay-offs may be measured in terms of units of money or time, chocolate, beer or anything that might be relevant to the situation. However, it
is often useful to generalise by writing pay-offs in terms of units of satisfaction or utility. Utility is an abstract, subjective concept and its use is widespread in economics. My utility from, say, a bar of chocolate is likely to be different from yours and anyway the two will not be directly comparable, but if we both prefer chocolate to pizza we will both derive more utility from the former. When a strategic situation is modelled as a game and the pay-offs are measured in terms of units of utility (sometimes called utils) then these will need to be assigned to the pay-offs in a way that makes sense from the player’s perspectives. What usually matters most is the ranking between different alternatives. Thus if a bar of chocolate makes you happier than a pizza the number of utility units assigned to the former should be higher. The actual number of units assigned will not always be important. Sometimes it is simpler not to assign numbers to pay-offs at all. Instead we can assign letters or symbols to pay-offs and then stipulate their rankings. For example, instead of assigning a pay-off of, say, ten to a bar of chocolate and three to a pizza, we could simply assign the letter A to the chocolate and the letter B to the pizza and specify that A is greater than B (i.e. \( A > B \)). This can be quite a useful simplification when we want to make general observations about the structure of a game.6 However, in some circumstances the actual value of the pay-offs is important and then we need to be a bit more precise (see Chapter 5).

Rational individuals are assumed to prefer more utility to less and therefore in a strategic game a pay-off that represents more utility will be preferred to one that represents less. Note that while this will always be true about levels of satisfaction or pleasure it will not always be the case when we are talking about quantities of material goods like chocolate – it is possible to eat too much chocolate. Players in a game are assumed to act rationally if they make plans or choose actions with the aim of securing their highest possible pay-off (i.e. they choose strategies to maximise pay-offs). This implies that they are self-interested and pursue aims. However, because of the interdependence that characterises strategic games, a player’s best plan of action for the game, their preferred strategy, will depend on what they think the other players are likely to do.

The theoretical outcome of a game is expressed in terms of the strategy combinations that are most likely to achieve the players’ goals given the information available to them. Game theorists focus on combinations of the players’ strategies that can be characterised as equilibrium strategies. If the players choose their equilibrium strategies they are doing the best they can given the other players’ choices. In these circumstances there is no incentive for any player to change their plan of action. The equilibrium of a game describes the strategies that rational players are predicted to choose when they interact. Predicting the strategies that the players in a game are likely to choose implies we are also predicting their pay-offs.

Games are often characterised by the way or order in which the players move. Games in which players move at the same time or their moves are hidden are called simultaneous-move or static games. Games in which the players move in some kind of predetermined order are call sequential-move or dynamic games. These two types of games are discussed in the following sections.
Pay-offs, equilibrium and rationality

- **Pay-off**: measures how well the player does in a possible outcome of a game. Pay-offs are measured in terms of either material rewards such as money or in terms of the utility that a player derives from a particular outcome of a game.

- **Utility**: a subjective measure of a player's satisfaction, pleasure or the value they derive from a particular outcome of a game.

- **Equilibrium strategy**: a ‘best’ strategy for a player in that it gives the player his or her highest pay-off given the strategy choices of all the players.

- **Equilibrium in a game**: a combination of players’ strategies that are a best response to each other.

- **Rational play**: players choose strategies with the aim of maximising their pay-offs.

1.3 Simultaneous-move games

In these kinds of games players make moves at the same time or, what amounts to the same thing, their moves are unseen by the other players. In either case, the players need to formulate their strategies on the basis of what they think the other players will do. We are going to look at three examples: hide-and-seek; a pub managers’ game; and a penalty-taking game. The first of these is a hidden-move game and the second and third are simultaneous-move games. Both types of games are analysed using the pay-off matrix or the strategic form of a game. In the first and third games the interests of the players are diametrically opposed; if one wins the other effectively loses. Games like this are games of pure conflict. Often the pay-offs of the players in games of pure conflict add to a constant sum. When they do the game is a constant-sum game. Both Hide-and-seek and the penalty-taking game are constant-sum games. If the constant sum is zero the game is a zero-sum game. Most games are not games of pure conflict. There is usually some scope for mutual gain through coordination or assurance. In such games there will be mutually beneficial or mutually harmful outcomes so that there are shared objectives. Games like this are sometimes called mixed-motive games. The pub managers’ game is a mixed-motive game.
1.3.1 Hide-and-seek

Hide-and-seek is played by two players called Robina and Tim. Robina chooses between only two available strategies: either hiding in the house or hiding in the garden. Tim chooses whether to look for her in the house or the garden. He only has 10 minutes to find Robina. If he looks where she is hiding (either the house or the garden) he finds her within the allotted time otherwise he does not. If Tim finds Robina in the time allotted he wins €50, otherwise Robina wins the €50.

Matrix 1.1 shows how the game looks from Robina’s perspective. The figures in the cells of the matrix are her pay-offs in euros. In the first cell of Matrix 1.1, on the top row of the first column, the zero shows that if Robina hides in the house and Tim looks in the house she loses. In the second cell, reading across the matrix, the 50 indicates that if she hides in the house and Tim looks in the garden she wins €50. On the bottom row of the matrix the 50 in the first column indicates that if Robina hides in the garden and Tim looks in the house she wins the €50 but the zero in the second column shows that if she hides in the garden and Tim looks in the garden she loses.

Matrix 1.1  Robina’s pay-offs in hide-and-seek

<table>
<thead>
<tr>
<th></th>
<th>look in house</th>
<th>look in garden</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Robina</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hide in house</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>hide in garden</td>
<td>50</td>
<td>0</td>
</tr>
</tbody>
</table>

Matrix 1.2 shows how the game looks from Tim’s perspective. In Matrix 1.2 the pay-offs in the cells show that if Robina hides in the house and Tim looks in the house he finds her and wins the €50, but if he looks in the garden when she hides in the house he loses. Similarly, if Robina hides in the garden and Tim looks in the house he loses but if he looks in the garden when she hides in the garden he finds her and wins the €50.

Matrix 1.2  Tim’s pay-offs in hide-and-seek

<table>
<thead>
<tr>
<th></th>
<th>look in house</th>
<th>look in garden</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Robina</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hide in house</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>hide in garden</td>
<td>0</td>
<td>50</td>
</tr>
</tbody>
</table>
To analyse the game we need to show both players’ pay-offs in the same matrix.\(^7\) This is done in Matrix 1.3 which is the strategic form or pay-off matrix of hide-and-seek. It shows all the possible pay-offs of the players that result from all their possible strategy combinations. It is a convention that in each cell the pay-off of the player whose actions are designated by the rows of the matrix are written first. The pay-offs of the player whose actions are denoted in the columns are written second. So in this pay-off matrix Robina’s pay-offs are written first and her pay-offs and strategies are highlighted in blue. For example, the pay-offs in the cell in the top row of the first column are 0 to Robina and 50 to Tim. This shows that if Robina hides in the house and Tim looks in the house, Tim wins the €50 and Robina’s pay-off is zero. The cell in the bottom row of the first column shows that if Robina hides in the garden and Tim looks in the house, Robina wins the €50 and Tim’s pay-off is zero.

Matrix 1.3 The pay-off matrix for hide-and-seek

<table>
<thead>
<tr>
<th></th>
<th>Tim</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>look in house</td>
<td>look in garden</td>
</tr>
<tr>
<td>Robina</td>
<td></td>
<td></td>
</tr>
<tr>
<td>hide in house</td>
<td>0, 50</td>
<td>50, 0</td>
</tr>
<tr>
<td>hide in garden</td>
<td>50, 0</td>
<td>0, 50</td>
</tr>
</tbody>
</table>

1.3.2 Pub managers’ game

In the pub managers’ game the players are two managers of different village pubs, the King's Head and the Queen's Head. Both managers are simultaneously considering introducing a special offer to their customers by cutting the price of their premium beer. Each chooses between making the special offer or not. If one of them makes the offer but the other doesn’t the manager who makes the offer will capture some customers from the other and some extra passing trade. But if they both make the offer neither captures customers from the other although they both stand to gain from passing trade. Any increase in customers generates higher revenue for the pub. If neither pub makes the discounted offer the revenue of the Queen’s Head is €7 000 in a week and the revenue to the Kings Head is €8 000. The pay-off matrix for this game is shown in Matrix 1.4 below which shows the pay-offs as numbers representing revenue per week in thousands of euros.
Matrix 1.4  Pay-off matrix for the Pub managers’ game

<table>
<thead>
<tr>
<th>Queen’s Head</th>
<th>special offer</th>
<th>no offer</th>
</tr>
</thead>
<tbody>
<tr>
<td>special offer</td>
<td>10, 14</td>
<td>18, 6</td>
</tr>
<tr>
<td>no offer</td>
<td>4, 20</td>
<td>7, 8</td>
</tr>
</tbody>
</table>

Following the convention already noted in section 1.3.1, the pay-offs of the player whose actions are designated by the rows are written first. So in this game the pay-offs of the manager of the Queen’s Head are written first and his strategies and pay-offs are highlighted in blue. The matrix shows that if the Queen’s Head manager makes the special offer his pay-off is 10 (i.e. €10000) if the King’s Head manager also makes the offer, and 18 if he doesn’t. Similarly if the King’s Head manager makes the offer his pay-off is 14 if the Queen’s Head manager also makes the offer, and 20 if he doesn’t.

Exercise 1.2

In the pub managers’ game what are the pay-offs of the managers if neither of them makes the offer? What is the pay-off of the Queen’s Head manager if he doesn’t make the offer but the manager of the King’s Head does? What is the pay-off of the King’s Head manager if he doesn’t make the offer but the manager of the Queen’s Head does?

Exercise 1.3

What do you think will be the outcome of the pub managers’ game. What do you think the managers will do?

Give some thought to Exercise 1.3. Although we haven’t actually looked at how to solve games yet, the pub managers’ game has an equilibrium that you can probably work out just by using a little common sense. In Chapter 2 you will see how to solve games like this in a systematic way. You will then be able to check whether your intuition was correct.

In hide-and-seek and the pub managers’ game the pay-offs represent monetary sums and it was convenient to do this. But this won’t always be possible as the next game shows.
1.3.3 Penalty taking

In the penalty-taking game the two players are the striker taking the penalty and the goalkeeper. Let’s assume that it is the last minute of the game and the score is one all. If the striker scores his team will win the game and if the goalkeeper saves the penalty his team will secure an honourable draw. If the striker scores he will be covered in glory and if the goalkeeper saves the penalty it will be he who is covered in glory. This time the pay-offs cannot really be measured in terms of money – being covered in glory is not really quantifiable in this way. Instead the pay-offs are best represented in terms of levels of subjective satisfaction or utility.

We can assume that if the striker misses, his satisfaction level is zero and if he scores, the goalkeeper’s satisfaction level is zero. This is clearly a simplification. You might prefer to assign a negative score in these circumstances or even different low scores. You can do this but bear in mind that these scores are subjective representations and therefore the players’ pay-offs are not directly comparable, even if we wanted to make this kind of comparison, which we don’t. If the striker scores, his satisfaction level will be sky-high and similarly, the goalkeeper will feel sky-high if he saves the penalty. How do we record these sky-high satisfaction levels? Well here, what really matters is the ranking of the players’ pay-offs so we could arbitrarily assign them a value of anything between 1 and some incredibly high figure like 100 billion. But smaller numbers are easier to handle so here I will use a pay-off of 10 to represent sky-high utility. You may prefer to add a few noughts and you should feel free to do that. You might also prefer to allocate different scores between the players for sky-high utility – perhaps you think the striker will feel happier if he scores than the goalkeeper will if he saves the penalty. But remember the scores are not directly comparable so this would really be an unnecessary complication.

In order to construct the pay-off matrix that corresponds to these pay-offs we need to make some additional assumptions. First of all we can assume that the striker always kicks the ball on target so he either scores or the goalkeeper makes a save. Second we can simplify the players’ strategies by assuming that the striker can only kick to his right, his left or straight ahead, these are his strategy choices. Similarly the goalkeeper can only move to the striker’s left, his right or he can stand his ground in the centre of the goal. If the goalkeeper’s action mirrors the striker’s he saves the penalty otherwise the striker scores. With these pay-offs and simplifying assumptions the pay-off matrix for this penalty-taking game looks like the one in Matrix 1.5 (I have highlighted the strategies and pay-offs of the striker).
Notice that in the cells of Matrix 1.5 the pay-offs always add to the constant sum 10 since if one player's pay-off is 10 the other's is zero. Therefore the interests of the players, like those of Robina and Tim in hide-and-seek, are diametrically opposed (in hide-and-seek the equivalent constant sum is 50). In both these games there is only one winner and the other player is a loser. Games like penalty-taking and hide-and-seek are called constant-sum games. If the constant sum in question is zero then the game is a zero-sum game. But any constant-sum game can be represented as a zero-sum game by subtracting half the constant sum from every pay-off. To see this subtract 5 from all the pay-offs in Matrix 1.5 or 25 from all the pay-offs in Matrix 1.3. All constant or zero-sum games are games of pure conflict and their outcomes are sometimes difficult to predict (you will see why in Chapter 2, Section 2.4.3). However, games of pure conflict won’t always be constant-sum games although they can usually be represented in this way.8

In the penalty-taking game left, centre and right are the pure strategies of the striker and the goalkeeper. If the striker decides that he is going to kick the ball to the left this would imply that he had chosen one of his pure strategies. Alternatively he might prefer to randomise between his pure strategies by, for instance, mentally throwing a dice before he runs up to kick the ball (or actually throwing a dice before running onto the pitch). He could kick to the left if the dice showed a 1 or a 2, to the right if it showed a 3 or a 4 and to the centre of the goal otherwise. If he did this the probability of him choosing any one of his three pure strategies would be \( \frac{1}{3} \). We could write this as \( \left( \frac{1}{3}, \text{left}; \frac{1}{3}, \text{centre}; \frac{1}{3}, \text{right} \right) \). Strategies that mix up a player's pure strategies in this way are called mixed strategies. Mixed strategies like these can be useful in games of pure conflict.
conflict like penalty taking, where one player doesn’t want the other to be able
to predict their move. Mixed strategies are explained in more detail in Chapter 6.

**Mixed strategy**
- A mix of pure strategies determined by a randomisation procedure.

In each of the games we have looked at so far we have used numbers to repre-
sent the players’ pay-offs. If the ranking of the pay-offs is all that matters (as
opposed to their absolute values) it is sometimes more convenient to write the
players’ pay-offs as letters. Using letters means that actual numbers do not have
to be assigned to pay-offs and this can be useful if you want to generalise the
results of one piece of analysis to other similar but not identical games. This
will be something that we will want to do in many of the chapters of this book
(see for example Chapter 3, Section 2). In the penalty game we could generalise
the pay-offs in this way by substituting the letter \( W \) for the number 10 on the
assumption that \( W > 0 \). Although the resulting game in
Matrix 1.5.1 looks a bit different from the one in Matrix 1.5, in all important
respects it is the same since \( W > 0 \) (as noted beneath the matrix). The striker
still prefers outcomes in which his chosen strategy is not matched by the goal-
keeper and the opposite is true for the goalkeeper.

**Matrix 1.5.1** Taking a penalty 1 with non-numerical pay-offs

<table>
<thead>
<tr>
<th></th>
<th>goalkeeper</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>left</td>
</tr>
<tr>
<td>left</td>
<td>0, ( W )</td>
</tr>
<tr>
<td>centre</td>
<td>( W, 0 )</td>
</tr>
<tr>
<td>right</td>
<td>( W, 0 )</td>
</tr>
</tbody>
</table>

\( W > 0 \)

**1.4 Sequential-move or dynamic games**

In sequential-move games players make moves in some sort of order. This
means one player moves first and the other player or players see the first
player’s move and can respond to it. Some illustrative examples are:
A firm considering entry into a monopolised industry where the incumbent may start a price war if it does enter.  

Chess.  

A series of offers and counter offers made by a potential buyer and seller of a house.  

A large firm, Apex, considering whether to launch an expensive advertising campaign which may be matched by its main rival, Convex.  

The leader of one country planning to invade another country.  

A film star who is deciding whether or not to sue a newspaper.  

A landowner who puts up a sign threatening to sue trespassers.

In each of these examples one of the players moves first and another sees the first player's move before deciding how to respond. This means that the order of moves is important and the analysis of this type of game has to take this into account. It is not always easy to do this using pay-off matrices and therefore sequential games are usually analysed using game trees or extensive forms like the one in Figure 1.1.

Figure 1.1 is drawn to represent the example in (iv). In this version of that game the two firms, Apex and Convex, choose between launching an advertising game or not. Apex moves first but the success of Apex’s campaign depends on what Convex does. A, C₁ and C₂ represent the decision points in the game. Apex’s choices are represented by the two branches that are drawn coming from the decision point or node labelled A. As Apex moves first this point is the first decision point in the game, the first point at which any player makes a move. At this point Apex chooses between launch or not launch. Whatever Apex decides Convex sees Apex’s move and can respond. If Apex launches its campaign the game moves to C₁ where Convex decides whether to launch its
campaign or not knowing full well that Apex has launched its campaign. At $C_1$ Convex can respond aggressively by launching its own campaign or respond passively by doing nothing. If Apex decides not to launch the campaign then the game moves to $C_2$ where Convex decides whether to launch its own expensive advertising campaign or not.

The pay-offs represent the firm’s profits in thousands of euros and they are written on the far right of the diagram at the endpoints or terminal nodes of the game tree, with Apex’s pay-offs written first. It is a convention that the pay-offs are written in the same order as the players’ moves, i.e. the pay-off of the player who moves first, in this case Apex, is written first. The pay-offs will always be written next to the terminal nodes of the appropriate branches of the game tree that mark the end of the game. In this game Apex’s pay-off depends not only on its own initial move but also on Convex’s response. Convex’s pay-off similarly depends on Apex’s initial move as well as its own move at either $C_1$ or $C_2$. If Convex responds aggressively to Apex’s move, whatever it is, by launching its own campaign Apex’s profits will be lower than if Convex had not launched its campaign. But if Apex does launch its campaign and Convex responds aggressively Convex’s profits are also lower as Convex’s action throws both firms into a damaging advertising war. However, if Apex doesn’t launch its campaign Convex benefits most by launching its campaign. This is shown by the players’ pay-offs at the ends of branches of the game tree. To see this look at the player’s pay-offs. When Apex decides on launching the campaign, if Convex responds by launching its own campaign, Apex’s pay-off is 2 and so is Convex’s. But if Convex doesn’t launch its own campaign both firms are better off – Apex’s pay-off is 6 and Convex’s is 3.

**Exercise 1.4**

What is Apex’s pay-off if it doesn’t launch the campaign but Convex does? What is Convex’s pay-off in these circumstances?

**Exercise 1.5**

What is Apex’s pay-off if it doesn’t launch the campaign and Convex doesn’t either? What is Convex’s pay-off in these circumstances?
The answer to Exercise 1.6 is not obvious but it is worth having a think about. In Chapter 4 we will use extensive forms like the one in Figure 1.1 to resolve sequential-move games like this game. You will be then be able to check whether your intuition was correct.

### 1.5 Repetition

Games that are only played once by the same players are called one-shot, single-stage or unRepeated games. Games that are played by the same players more than once are known as repeated, multi-stage or n-stage games where n is greater than one. The strategies of the players in repeated games need to set out the moves they plan to make at each repetition or stage of the game. These kinds of strategies are called meta-strategies.

The penalty game is a game that is likely to be played by the same players more than once; the same players in teams tend to take the penalties. Suppose the penalty game in Matrix 1.5 was played six times by the same two players. The striker’s meta-strategy for this repeated game could be to kick to the left in the first two repetitions then to the centre of the goal then twice to the right and then to the centre again. We would write this as (left, left, centre, right, right, centre). Alternatively he could choose a mixed strategy by randomising between left, right and centre every time he went to kick the ball. If his mixed strategy prescribed that he played each of his pure strategies with a probability of one-third then over the course of the repeated game we would expect to see him kicking to the left, right and centre a third of the time each. Repeated games are analysed in Chapter 8 and in some of the repeated games we are going to look at the players play mixed strategies.

### 1.6 Cooperative and non-cooperative games

Whether a game is cooperative or not is a technical point. Essentially a game is cooperative if the players are allowed to communicate and any agreements they make about how to play the game as defined by their strategy choices are enforceable. Most of the games we will look at in this book are non-cooperative.
even though in some of them players choose between cooperating with each other or not (for example in the prisoners’ dilemma games in Chapter 3). But being able to choose to cooperate does not make a game cooperative in the technical sense as such a choice is not necessarily binding. Being able to enforce agreements makes the analysis of cooperative games very different from that of non-cooperative games. Because agreements can be enforced the players have an incentive to agree on mutually beneficial outcomes. This leads cooperative game theory to focus on strategies that are implemented in the players’ joint or collective interests. This is not the case in non-cooperative game theory where it is assumed that player’s act only in their own self-interest. Some bargaining games are cooperative in this technical sense and these as well as non-cooperative bargaining games are analysed in Chapter 9.

1.7 N-player games

N is the number of players in the game. If a game has two players then it is a 2-player game. But if there are more than two players then the game is an N-player game where N is greater than 2. Most of the games we will look at in this book are 2-player games. The greater the number of players involved in a game the more complex it is likely to be.

1.8 Information

The equilibrium strategies of the players will depend on what kind of information players have about each other. In some games players will be very well informed about each other but this will not be true in all games. The information structure of a game can be characterised in a number of ways (see, for example, Montet and Serra, 2003: 4–6). The categories used in this book are perfect information, incomplete information and asymmetric information. If information is perfect then each player knows where they are in the game and who they are playing. If information is incomplete then a pseudo-player called ‘nature’ or ‘chance’ moves in a random way that is not clearly observed by all or some of the players. If not all the players observe the chance move then the information is also asymmetric. When information is asymmetric not all players have the same information. Instead some player has private information.

In all the games in Chapters 2–4 the players have perfect information. This is unlikely in real life and if game theory is to be really useful it needs to incorporate imperfect information. You will see how to do this in Chapters 5, 6 and 7.
In the games analysed in these chapters one or more of the players is less than perfectly informed.

When information is not perfect there is uncertainty in one or more of the players’ minds about where they are in a game or who they are playing. For the players this implies an extra element of risk. In risky situations the outcome is uncertain and this uncertainty is characterised by a probability distribution. In strategic games risk is incorporated in terms of the initial or prior beliefs of the players. In some situations the players may also be able to update their beliefs as and when they receive information (see Chapter 7). Risk is not unique to strategic games. It is also a feature of many situations where an individual’s choice of action is not strategically related to that of anyone else. In these cases risk is non-strategic. You will see how to model non-strategic risk in Chapter 5.

Whether the situation is strategic or not, where risk is involved decision makers need to incorporate the relevant probabilities into their decision making. They do this by forming expectations about likely outcomes and rational decision makers are assumed to choose in order to maximise their expected pay-off. This is an average of all the possible pay-offs corresponding to a given choice. It is calculated by multiplying (or weighting) each pay-off by the probability that it will occur. If the pay-offs are written as units of money or even chocolate then this calculation generates an expected value in terms of either money or chocolate. If the pay-offs are written in terms of utility values then the calculation generates an expected utility. These two alternatives are discussed further in Chapter 5 but for the moment it may be helpful to note that expected utility is potentially the more useful measure as it can incorporate people’s different attitudes to risk.

Summary

In this chapter you have learned about some of the basic ideas and concepts that are central to game theoretic analysis. Games and game theory were defined in terms of strategic interdependence and some game theoretic terminology was explained. You have seen that games can be divided into two main groups according to whether they involve simultaneous or sequential moves. Simultaneous-move games are represented using pay-off matrices or strategic forms. Sequential-move or dynamic games are usually represented by extensive forms or game trees. Simultaneous-move and sequential games can be played only once or they can be repeated. In the next two chapters you will learn how to model and predict outcomes in single-stage simultaneous-move games. Sequential-move games are analysed in Chapter 4. In Chapters 5 and 6 single-stage games with incomplete information are analysed. Repeated games are the subject of Chapter 8. Strategic games can be either non-cooperative or cooperative. Most of the games you will see in this book are non-cooperative. Cooperative games are considered in Chapter 9.
1.1
There are no explicitly right or wrong answers in this exercise. By way of an example an answer for (e) might go as follows: in the civil war the players are the two opposing factions. At least one of the factions needs to compromise in order for an agreement to be reached. If only one party compromises (or compromises more than the other) they lose out in the agreement but if neither compromises there will be no agreement and the war will continue (to the disadvantage of both). Interestingly scientists at the Santa Fe Institute in New Mexico have devised a game that models a scenario a bit like this to calculate how the probability of each party’s decision to fight in a civil conflict or to compromise changes as the terms of the proposed agreement change (Dispatch report, Guardian, 18 November 2003).

1.2
If neither manager makes an offer the manager of the Queen’s Head gets 7 and the manager of the King’s Head gets 8. If the Queen’s Head manager doesn’t make the offer but the manager of the King’s Head does the manager of the Queen’s Head gets 4. If the King’s Head manager doesn’t make the offer but the manager of the Queen’s Head does the manager of the King’s Head gets 6.

1.3
Both managers are better off making the offer whatever the other manager does so there is no reason to expect them not to make the offer. The most likely outcome seems to be that both managers will make the offer. This is actually the dominant strategy equilibrium of the game as you will see in Chapter 2.

1.4
In these circumstances Apex’s pay-off will be 3 and Convex’s pay-off will be 6.

1.5
In these circumstances Apex’s pay-off will be 4 and Convex’s pay-off will be 4.

1.6
If Apex launches it gets either 6 if Convex doesn’t launch or 2 if Convex does. As Convex gets 3 by not launching if Apex also launches but 2 otherwise it should not launch if Apex also launches. Apex is assumed to know this. If Apex doesn’t launch it gets at most 4. So if Apex believes that if it launches Convex will not launch Apex should launch. Don’t worry if this chain of logic is not altogether clear as sequential games like this will be examined in detail in Chapter 4.
Think of one or two examples of real-life situations that could be represented as games and describe them using game theoretic terminology such as player, pay-offs and strategies.

In the examples you have thought of, do the players move simultaneously or sequentially and are their moves hidden or seen?

What is meant by strategic interdependence?

How can player’s pay-offs in games be represented?


Schelling (1960: 150) defines a strategic game in terms of dependence of one person’s choice of action on what he expects another to do and a strategic move as an action by one person that influences another person’s choice by affecting their expectations of how the first person will behave.

Except for some very expensive luxury items and some necessities, the relationship between consumer demand and price is assumed to be negative, i.e. if price rises demand falls and vice versa. Thus to encourage more sales in an industry market prices need to fall (assuming that no other important factors, for instance advertising or consumer tastes, change).

Binmore (1990) distinguishes three additional purposes of game theoretic models: description, investigation and prescription.

The thinking and rationality assumptions are not always applicable in evolutionary games (see Chapter 6).

If you want to know more about utility most introductory economics and all intermediate microeconomic textbooks have a chapter explaining how the concept is used to analyse various types of human behaviour (see, for example, Dawson, 2001, Chapter 4 in Himmelweit et al., 2001).

Or bimatrix as there is more than one pay-off in each cell.

Since in zero-sum games the pay-off of one player is just the negative of the other’s, pay-off matrices for zero-sum games often only show the pay-offs of one of the players.

An oligopoly market dominated by only two large firms is called a duopoly.
MOoving together

Concepts and techniques

- Simultaneous moves
- Dominant strategies and dominated strategies
- Dominant-strategy equilibrium
- Iterated-dominance equilibrium
- Nash equilibrium
- Pareto efficiency
- Assurance games.

After working through this chapter you will be able to:

- Analyse games in which the players move simultaneously or their moves are hidden
- Explain what is implied by a dominant strategy
- Determine the dominant-strategy equilibrium of a game if one exists
- Find the iterated-dominance equilibrium of a game if it has one
- Explain what is implied by the concept of a Nash equilibrium
- Determine whether a game has a Nash equilibrium
- Demonstrate that a dominant-strategy equilibrium is also a Nash equilibrium
- Show that some games have more than one Nash equilibrium and some games have none.
In this chapter you are going to learn how to analyse games in which the players choose their strategies and make their moves at the same time or their moves are hidden from each other. Games of this kind can be analysed in the same way. They are called simultaneous-move, static- or hidden-move games. You saw some examples of these kinds of games in Section 1.3 of Chapter 1. In the penalty-taking game and the pub managers’ game the players moved simultaneously. In hide-and-seek Robina’s move was, literally, hidden. Another example of a hidden move game is voting in an election where voters’ choices are made in secret. In general elections, which can last several days, voters are kept deliberately uninformed about how others are voting by laws that prohibit the results of exit polls being revealed until polling is closed. In some countries polls are also prohibited for a few days leading up to an election. In games like this where the players’ moves are hidden from each other and in games where the players move simultaneously a player’s choice cannot be made contingent on another player’s actions. Players therefore need to reason through the game from their own and the other players’ perspectives in order to make a rational choice.

An underlying assumption of game theoretic models that enables players to carry out these kinds of thought processes is that they possess common knowledge that the other players are rational. This means that each player aims to choose a strategy that will secure their highest possible pay-off in the full knowledge that all the other players are trying to do exactly the same thing. The players will only be satisfied with their strategy choices if they are mutually consistent. By this I mean that no player could have improved their pay-off by choosing a different strategy given the strategy choices of the other players. If the strategy choices of the players are mutually consistent in this way then none of the players has an incentive to make a unilateral change. In these circumstances the strategies of the players constitute an equilibrium. However, the precise nature of the equilibrium will depend on the game in question. The main equilibrium concepts used to resolve simultaneous-move games are those of a dominant-strategy equilibrium, an iterated-dominance strategy equilibrium and a Nash equilibrium. We are going to consider each of these in turn.

2.1 Dominant-strategy equilibrium

In a dominant-strategy equilibrium every player in the game chooses their dominant strategy. A dominant strategy is a strategy that is a best response to
all the possible strategy choices of all the other players. A game will only have a
dominant-strategy equilibrium if all the players have a dominant strategy. To
understand what this means we are going to look at a number of examples in
detail. The first of these is the pub managers’ game that you saw in Chapter 1,
Section 1.3.2.

2.1.1 PUB MANAGERS’ GAME

The players in this game are the two managers of different village pubs, the
King’s Head and the Queen’s Head. They are simultaneously considering
making a special offer to their customers. The strategic form of the pub man-
gagers’ game is reproduced here as Matrix 2.1. As before, the pay-offs represent
revenue per week in thousands of euros and to help you with the analysis that
follows the strategy choices and the pay-offs of the Queen’s Head manager are
highlighted in colour.

<table>
<thead>
<tr>
<th>King’s Head</th>
<th>special offer</th>
<th>no offer</th>
</tr>
</thead>
<tbody>
<tr>
<td>special offer</td>
<td>10, 14</td>
<td>18, 6</td>
</tr>
<tr>
<td>no offer</td>
<td>4, 20</td>
<td>7, 8</td>
</tr>
</tbody>
</table>

In the pub managers’ game there are four possible strategy combinations corre-
sponding to four possible sets of pay-offs:

1. The Queen’s Head does not make the special offer and neither does the
   King’s Head: neither pub gains custom. The pay-off to the Queen’s Head is 7
   and the pay-off to the King’s Head is 8.

2. The Queen’s Head makes the special offer and so does the King’s Head: both
   pubs gain customers. The pay-offs are 10 to the Queen’s Head and 14 to the
   King’s Head.

3. The Queen’s Head makes the offer but the King’s Head does not: the Queen’s
   Head gains custom and the King’s Head loses custom. The pay-offs are 18 to
   the Queen’s Head and 6 to the King’s Head.

4. The Queen’s Head does not make the special offer but the King’s Head does:
   the Queen’s Head loses custom but the King’s Head gains custom. The pay-
   offs are 4 to the Queen’s Head and 20 to the King’s Head respectively.

To see if the game has a dominant-strategy equilibrium we need to check
whether both players have a dominant strategy. In this game they do. First let’s
consider the game from the perspective of the manager of the Queen's Head. If the Queen's Head manager makes the special offer his pay-off is either 10 or 18. It is 10 if the manager of the King's Head also makes the offer and 18 if he doesn't. If the manager of the Queen's Head doesn't make the offer then his pay-off is either 4 or 7. It is 4 if the manager of the King's Head also makes the offer. This is less than the 10 he would have got if he had introduced the offer. If he doesn't make the offer and neither does the manager of the King's Head then his pay-off is 7 which is also less than the 18 he would have got if he had introduced the offer in these circumstances.

This reasoning shows that the manager of the Queen's Head is better off if he makes the special offer whatever the manager of the King's Head does. Thus making the offer is a dominant strategy for him; it is a best response to whatever the manager of the King's Head decides to do. Take another look at Matrix 2.1. Each of the Queen's Head's pay-offs in the top row is higher than the corresponding pay-off in the same column of the bottom row (10 is greater than 4 and 18 is greater than 7). This shows that making the special offer is a dominant strategy of the manager of the Queen's Head.

Similar reasoning can be applied to the strategy choices of the manager of the King's Head to show that introducing the offer is also a dominant strategy for him. Visually this is clear in the matrix because each of the King's Head's pay-offs in the first column on the left is higher than the corresponding pay-off in the same row of the second column on the right (14 is greater than 6 and 20 is greater than 8). Since making the offer is a dominant-strategy for both pub managers the dominant strategy equilibrium of this game is for both managers to make the special offer. This equilibrium is written as \{special offer, special offer\}. When the equilibrium is written in this way it is conventional to write the equilibrium strategy of the player whose strategies are denoted in the rows of the pay-off matrix first. However in this game the equilibrium strategies of the players are the same and the order in which their strategies are written is not really an issue but this will not always be the case. At this point it would be a good idea to check your answer to Exercise 1.3. Was your intuition correct? Can you explain how you rationalised the answer you gave (whatever it was)?

The idea of a dominant-strategy equilibrium is an important one. In a dominant-strategy equilibrium all the players pick their dominant strategies, their best responses to all the available strategies of all the other players. If a player has a dominant strategy and he wants to maximise his pay-off then there is no reason to believe that he will choose anything else. Thus if a game has a dominant-strategy equilibrium we can be fairly bullish about predicting this as the likely outcome as long as the players are rational and the game captures all the salient aspects of the situation under examination. Unfortunately, games with dominant strategy equilibria may not be that common in real life although you are going to see a few more that do.
2.1.2 Labour market legislation

In this game represented in Matrix 2.2 the governments of two neighbouring countries, Jesmania and Rosatia, are considering imposing new labour market legislation, e.g. legislation relating to health and safety, paternity rights or minimum wages. Polls have predicted that the new legislation will be a net vote winner for the government. This is true even though some votes are expected to be lost because firms have threatened to lay off workers due to the higher labour costs associated with complying with the legislation. The net gain in votes is expected to be much higher if the neighbouring country imposes similar legislation. But if the neighbouring country does not impose legislation their labour will be cheaper and some firms from the first country are expected to relocate to the second. This means more jobs will be lost in the country that introduces the legislation and the net gain in votes will be much smaller. The pay-offs in Matrix 2.2 are expected gains in millions of votes. As in the pub managers’ game, the strategy choices and the pay-offs of the player whose strategies are given in the rows of the matrix, the Government of Jesmania, are highlighted.

Matrix 2.2 Legislation game

<table>
<thead>
<tr>
<th>government of Jesmania</th>
<th>government of Rosatia</th>
</tr>
</thead>
<tbody>
<tr>
<td>legislate</td>
<td>don’t legislate</td>
</tr>
<tr>
<td>legislate</td>
<td>5, 5</td>
</tr>
<tr>
<td>don’t legislate</td>
<td>0, 1</td>
</tr>
</tbody>
</table>

In the legislation game the dominant strategy of both governments is to introduce the legislation. If Jesmania introduces the legislation and Rosatia does not
then the expected number of votes gained by the government of Jesmania is only 1 million. But if Rosatia also introduces the legislation the government of Jesmania stands to gain 5 million extra votes. However, if the government of Jesmania doesn’t introduce the legislation its votes do not increase at all; doing nothing achieves nothing. Thus to legislate is a dominant strategy for Jesmania, as it is for Rosatia, and the dominant-strategy equilibrium of this game is {legislate, legislate}.

You can see this in the matrix by comparing the pay-offs of the Jesmanian government on the top row with those on the bottom row. Each of the Jesmanian government’s pay-offs on the top row is higher than the corresponding pay-off in the same column on the bottom row (5 is greater than 0 and 1 is greater than 0). Similarly each of the Rosatian government’s pay-offs in the first column on the left are higher than the corresponding pay-off in the second column on the right.

However, if more firms were to relocate than expected as a result of only one country imposing the legislation then the actual net gain from legislation could be negative. In these circumstances legislation would not be a dominant strategy for either government. It is this kind of possibility, an aspect of globalisation, that can prevent governments introducing what many voters believe to be sensible legislation.

**Dominant-strategy equilibrium**

- **Strong dominant-strategy equilibrium**: a combination of strongly dominant strategies; in a two-player game a pair of strategies that for each player are strictly best responses to all of the strategies of the other player.

- **Weak dominant-strategy equilibrium**: combination of dominant strategies where some or all of the strategies are only weakly dominant.

**2.1.3 Port access**

Before turning to the derivation of iterated-dominance equilibria we are going to look at one more example of a game with a dominant-strategy equilibrium. The port-access game is a simplified version of an example in Gates and Humes (1997: Chapter 2). The game represents a series of interactions in 1985 between the USA and New Zealand in connection with an ANZUS (Australia, New Zealand, United States) alliance exercise. In the simplified port-access game described here one country, the USA, requires port access for its naval vessels in the other country, New Zealand. However, New Zealand has declared itself a nuclear-free zone and may refuse to allow access unless the USA gives a
guarantee that they are not carrying any nuclear weapons. Alternatively it can simply allow access to American ships without question. For security reasons the USA is unable to give any guarantees about their vessels’ weaponry. In response to New Zealand’s nuclear-free stance it can simply acquiesce by putting into harbour somewhere else or retaliate by declaring the alliance between the two countries void. The latter is assumed to be a costly alternative for both countries but more so for the USA.

The strategic form for the port-access game is shown in Matrix 2.3. As only the ranking of the pay-offs to the players is important and it is difficult to conceive of meaningful numerical equivalents for the utilities of the two governments, the pay-offs are delineated as letters. The pay-offs of the USA are A, B, C and D where A > B > C > D. New Zealand’s pay-offs are a, b, c and d where a > b > c > d.

Matrix 2.3 Port access

<table>
<thead>
<tr>
<th></th>
<th>allow access</th>
<th>refuse access</th>
</tr>
</thead>
<tbody>
<tr>
<td>maintain alliance</td>
<td>A, b</td>
<td>C, a</td>
</tr>
<tr>
<td>void alliance</td>
<td>B, d</td>
<td>D, c</td>
</tr>
</tbody>
</table>

For the USA: A > B > C > D
For New Zealand: a > b > c > d

New Zealand’s highest pay-off, a, is earned when it maintains the confidence of its electorate by refusing access to American vessels but the USA maintains the alliance. The worst possible outcome for New Zealand (leading to a pay-off of d) arises if it allows access to USA vessels, losing the trust of its electorate, but the USA still voids the alliance. For the New Zealand government it is better to refuse access to USA vessels whatever the USA does since a > b and c > d. Refusing access is therefore a dominant strategy for New Zealand.

The USA’s highest pay-off, A, is earned when New Zealand allows access and the USA maintains the alliance. The worst outcome for the USA (leading to a pay-off of D) arises if New Zealand refuses access and it retaliates by voiding the alliance. The alliance is important to the USA and therefore it is always better to maintain the alliance whatever New Zealand does.

Because A > B and C > D maintaining the alliance is a dominant strategy for the USA. To be more precise it is a strictly dominant strategy. If either of these inequalities were equalities then maintaining the alliance would only have been a weakly dominant strategy for the USA. As a > b and c > d refusing entry is similarly a strictly dominant strategy for New Zealand (if either inequality were an equality then refusing entry would be a weakly dominant strategy). If both players choose their dominant strategies they will have no incentive to deviate from them. The dominant-strategy equilibrium of this game is therefore {maintain the alliance, refuse access}. 
When the real-life version of this game was played out New Zealand refused entry but the USA retaliated by withdrawing from military relations with New Zealand. Why then was the dominant-strategy equilibrium not played out? Well, assuming the governments of the USA and New Zealand acted rationally the answer to this question must be that the port-access game described here does not fully capture all the salient features of the game that was actually played out. This is a valid criticism. First of all in the real-life version of this game the players did not move simultaneously – New Zealand moved first. In many, if not most strategic games the order of moves matters and as we shall see in Chapter 4, when sequential moves are incorporated the equilibrium of a game can change. Second, the pay-offs of the USA do not take into account how its other military relationships might be damaged by acquiescence. Doing so could change the pay-offs considerably even to the extent that maintaining the alliance is no longer a dominant strategy for the USA. Lastly, the port-access game may only have been one stage of a repeated game being played out by the USA with its allies. Modelling a game as a repeated game allows reputation effects to be incorporated and these can also change the predicted outcome as we shall see in Chapter 8. These kinds of considerations should alert you to some of the dangers implicit in game theoretic modelling. By necessity game theory simplifies and abstracts from the real world in order to develop hypotheses about how the real world works. But if important elements are left out in the modelling process little of consequence may be learned. It is therefore of considerable importance, when constructing game theoretic models, to try to capture as many of the salient features of the game’s real-life counterpart as possible.

**Exercise 2.1**

The players in the foreign investment game represented in Matrix 2.4 are two large firms, Art and Bart, that monopolise a domestic market. The firms are independently deciding whether to invest in new outlets abroad or not. The new investments cost money but open up new foreign markets thereby generating higher profits. If only one firm invests abroad it claims all the available foreign markets. If both of the firms invest in new outlets the foreign markets are shared. Each firm has to decide whether to make the foreign investments or not without knowing the other firm’s choice. The pay-offs in Matrix 2.4 reflect the utilities of the firms’ directors from the profits the firms make on the assumption that higher profits generate more utility. What is the dominant strategy equilibrium of this game? (Hint: ask yourself what is the preferred strategy of Art, the one that is a best response to whatever firm Bart chooses. Then ask yourself the same question about Bart. If it helps you can highlight the pay-offs of one of the players as I did in the pub managers’ game and the legislation game.)
Many games do not have a dominant-strategy equilibrium. In this case we can look for an iterated-dominance equilibrium. A two-person game that doesn’t have a dominant-strategy equilibrium may have an iterated-dominance equilibrium if one of the players has either a strongly or weakly dominant strategy. A strongly dominant strategy is one where the pay-off to the player from choosing that strategy is better than that from any other strategy in response to any strategy the other player picks. A weakly dominant strategy is one where the pay-off
to the player from choosing that strategy is at least as good as any other strategy and better than some in response to whatever strategy the other player picks (see Section 2.3.1 for a more formal definition). If a player in a game has a dominant strategy all their other strategies are dominated strategies.

If one of the players in a two-player game has a dominant strategy then even if the other player doesn’t the game may still have an iterated-dominance equilibrium. An additional requirement when the player with the dominant strategy has a choice of only two strategies is that the other player has a best response to the dominant strategy of the first player. More generally in a two-person game an iterated-dominance equilibrium is a strategy combination where for at least one player their equilibrium strategy (i) is as good any other strategy and better than some in response to all the non-dominated strategies of the other player and (ii) is a best response to the equilibrium strategy of the other player. For the other player, their equilibrium strategy is a best response to the equilibrium strategy of the first player. This sounds complicated but these ideas will become clearer as we look at some examples. First we will look at games with a strong iterated-dominance equilibrium and then at games with only a weak iterated-dominance equilibrium.

2.2.1 Friends or enemies?

The game in Matrix 2.6 is a variation on the friends game in Exercise 2.2 but this version of the game doesn’t have a dominant-strategy equilibrium. In this game although Mr Column still has a preference for the party and still wants to be with Ms Row, she doesn’t want to be with him at all, quite the opposite. In this game Mr Column is Ms Row’s stalker. He wants to be with her but she doesn’t want to be anywhere near him. However, Mr Column’s preference for the party still makes going to the party a dominant strategy for him. But Ms Column doesn’t have a dominant strategy she just wants to avoid Mr Column by choosing the opposite of whatever he chooses. Because Ms Row doesn’t have a dominant strategy the game doesn’t have a dominant-strategy equilibrium. But it does have a strong iterated-dominance equilibrium.

Matrix 2.6  Friends or enemies

<table>
<thead>
<tr>
<th></th>
<th>party</th>
<th>club</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>party</strong></td>
<td>1, 3</td>
<td>2, 0</td>
</tr>
<tr>
<td><strong>club</strong></td>
<td>2, 2</td>
<td>1, 1</td>
</tr>
</tbody>
</table>
To find the strong iterated-dominance equilibrium of a game, if one exists, all that you need to do is delete strongly dominated strategies from the strategic form of the game until only a single pair of strategies remain. In the friends or enemies 1 game club is a dominated strategy for Mr Column because his pay-off from choosing party is always higher than his pay-off from choosing club. If Ms Row chooses party and Mr Column chooses party his pay-off is 3 but if he chooses club his pay-off is 0. Similarly, if Ms Row chooses club Mr Column's pay-off from choosing party is 2 but if he chooses club his pay-off is only 1. Consequently he always gets less by choosing club which means that club is a strongly dominated strategy for Mr Column (party is a strongly dominant strategy) so if he is rational he will never choose club. Why would he when he can always do better by going to the party? Since Mr Column will never choose club we can delete the column corresponding to his choice of club. This produces Matrix 2.6.1.

Matrix 2.6.1 Deleting Mr Column’s dominated strategy of club

<table>
<thead>
<tr>
<th>Ms Row</th>
<th>Mr Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>party</td>
<td>1, 3</td>
</tr>
<tr>
<td>club</td>
<td>2, 2</td>
</tr>
</tbody>
</table>

In the game in Matrix 2.6.1 club is a dominant strategy for Ms Row (she gets 2 by going to the club and only 1 by going to the party) so we can also delete the row corresponding to her option of party. This leaves only one strategy for each player; club for Ms Row and party for Mr Column. This pair of strategies is the strong iterated-dominance equilibrium of the game. We found it by deleting the players’ strongly dominated strategies, initially club for Mr Column and then party for Ms Row. This left only one pair of strategies: club for Ms Row and party for Mr Column, implying that the strong iterated-dominance equilibrium of the game is for Ms Row to go to the club and Mr Column to go to the party. This is written as {club, party} because, as noted above, it is conventional to write the equilibrium strategy of the player whose strategies are delineated by rows, Ms Row in this case, first.

Iterated-dominance equilibrium

- An equilibrium found by deleting strongly or weakly dominated strategies until only one pair of strategies remains.
2.2.2 Political ambition

Matrix 2.7 represents a game called political ambition. In this game the players are an incumbent member of parliament (the MP) who has a safe seat and a local councillor who is considering challenging the MP by standing for election herself. The incumbent MP is deciding between leaving office (resigning his parliamentary seat, perhaps to spend more time with his family) and standing for re-election. The MP enjoys his job and will only resign if an effective challenge can be mounted against him. The pay-offs represent the players’ utilities from the alternative outcomes. In this particular version of political ambition the incumbent MP is very popular with his constituents and is electorally invulnerable; in the pay-off matrix this is represented by his higher pay-offs (in the first column on the left of the matrix) from standing whatever the challenger does. Resign is consequently a strongly dominated strategy for the MP and we can delete the right-hand column from the matrix. But the challenger’s case is hopeless if the MP stands for re-election (this is represented by the challenger’s pay-off of –15 in the cell on the bottom row of the first, the left-hand column). Deleting challenge for the challenger leaves only one pair of strategies remaining: no challenge and stand. This strategy pair constitutes the strong iterated-dominance equilibrium of the game which we can write as \{no challenge, stand\}.

<table>
<thead>
<tr>
<th>Challenger</th>
<th>Incumbent MP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stand</td>
</tr>
<tr>
<td>No challenge</td>
<td>5, 10</td>
</tr>
<tr>
<td>Challenge</td>
<td>-15, 9</td>
</tr>
</tbody>
</table>

Exercise 2.3

What would the pay-offs in political ambition look like if the incumbent MP was electorally vulnerable to a challenge? Can you construct a pay-off matrix that illustrates this possibility? Does the game you have constructed have an iterated-dominance equilibrium or dominant-strategy equilibrium?

2.2.3 Friends or enemies again

The game represented in Matrix 2.8 is called friends or enemies. In this version of the friends or enemies game both players receive an invite to a wedding on the
same day as the party and the club event. Going to the wedding is a new option for the players and Mr Column prefers to go to the wedding if Ms Row also goes to the wedding. Consequently neither player has a dominant strategy. But will Ms Row ever want to go to the wedding? This seems unlikely as her pay-offs make going to the wedding a strongly dominated strategy. If Mr Column goes to the party she prefers the club, if he goes to the club she prefers the party and if he goes to the wedding she doesn’t care where she goes as long as it isn’t the wedding. She therefore has no reason to go the wedding and we can rule out wedding for Ms Row by deleting the bottom row of the matrix. Mr Column will never choose wedding in response to either party or club so we can rule out going to the wedding for him too. This leaves us with the original friends or enemies game represented in Matrix 2.6. You have already seen that the iterated-dominance equilibrium of that game was \{club, party\} hence the iterated-dominance equilibrium of friends or enemies 2 must also be \{club, party\}.

**Matrix 2.8  Friends or enemies 2**

<table>
<thead>
<tr>
<th></th>
<th>Mr Column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>party</td>
</tr>
<tr>
<td>party</td>
<td>1, 3</td>
</tr>
<tr>
<td>club</td>
<td>2, 2</td>
</tr>
<tr>
<td>wedding</td>
<td>1, 3</td>
</tr>
</tbody>
</table>

### 2.2.4 Friends or enemies 3

The game in Matrix 2.9 is another version of the friends or enemies game. It doesn’t have a strong iterated-dominance equilibrium but it does have a weak iterated-dominance equilibrium. In this version of the game Ms Row still does not want to meet Mr Column and Mr Column is still stalking Ms Row. However, Mr Column’s preference for the party is not as strong as it was. His pay-offs are such that if Ms Row goes to the party he also prefers to go the party but if she goes to the club he is indifferent between going to the club or the party. But by going to the club he risks ending up with nothing and since he can always do as well or better by going to the party why would he go to the club? There is no reason why he should and this is what makes club a weakly dominated strategy for Mr Column. We can therefore delete the column on the right corresponding to his choice of club. This makes club a dominant strategy for Ms Row and we can delete the top row of what’s left of Matrix 2.9. As the only remaining strategy pair this makes \{club, party\} the iterated-dominance equilibrium of the game. But now it is only a weak iterated-dominance equilibrium as club is only weakly dominated by party for Mr Column.
2.2.5 Battle of the Bismarck Sea

The game represented in Matrix 2.10 is another game with a weak iterated-dominance equilibrium. It is a classic example referred to in many game theory texts. The players are the Japanese Navy and the USAF (the US Air Force). The Japanese Navy are transporting troops across the Bismarck Sea and the USAF wants to bomb them. The Japanese Navy are choosing between two routes, a northern or a southern route. The USAF has to decide where to send their planes to look for the Japanese Navy. If the USAF initially send their planes along the wrong route they can send them back out on the other route but the opportunity for bombing will be reduced and less damage will be inflicted on the Japanese Navy. The northern route is shorter than the southern route and therefore the Japanese Navy is more vulnerable to attack along the latter (they can be bombed for longer).

The pay-offs in this game indicate that north is a weakly dominant strategy for the Japanese Navy; because the southern route is longer, choosing south is as, or more costly than, choosing north, even if the USAF initially chooses north. Eliminating south for the Japanese Navy leaves north as a dominant strategy for the USAF and the strategy combination \{north, north\} as the weak iterated-dominance equilibrium of this game. In the real-life version of this game played out in the South Pacific in March 1943 this was the actual outcome.² Note that the battle of the Bismarck Sea is also a zero-sum game: the USAF’s gain is the Japanese Navy’s loss.
2.2.6 Weak iterations

Not every game that doesn’t have a dominant-strategy equilibrium has an iterated-dominance equilibrium and more worryingly some games may have more than one weak iterated-dominance equilibrium. This possibility suggests that the concept of a weak iterated-dominance equilibrium may not be quite as useful as its name suggests. If there is more than one iterated-dominance equilibrium in a game how can any of them encapsulate the idea of dominance in a meaningful way?

To see what is implied in this kind of situation take a look at the abstract game represented in Matrix 2.11. In the weak iterations game the players choose between three alternative strategies and there are two weak iterated-dominance equilibria: {middle, centre} and {down, left}.

**Matrix 2.11 \ Weak iterations**

<table>
<thead>
<tr>
<th></th>
<th>Player B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>left</td>
</tr>
<tr>
<td>up</td>
<td>3, 6</td>
</tr>
<tr>
<td>middle</td>
<td>2, 1</td>
</tr>
<tr>
<td>down</td>
<td>3, 7</td>
</tr>
</tbody>
</table>

To find the first iterated-dominance equilibrium in weak iterations, {middle, centre} you can delete down as this is a weakly dominated strategy for A. This makes left a strongly dominated strategy for B. Deleting left makes up a weakly dominated strategy for A. Deleting A’s strategy of up leaves centre as a dominant strategy for B making {middle, centre} the iterated-dominance equilibrium. Alternatively, delete middle which is also weakly dominated for A, then centre, now strongly dominated for B. This leaves down as weakly dominant for A and deleting up leaves left as a dominant strategy for B making {down, left} the iterated-dominance equilibrium of the game. The problem here is a theoretical one, that there is more than one weakly dominated strategy and the order of deletion matters. This is something to be aware of when using the method of deleting weakly dominated strategies to find the equilibrium of a game. Evidence from experiments\(^3\) also suggests that there are some descriptive limitations of the iterative method. For example, Beard and Beil (1994) tested the willingness of players to choose their iterated dominance strategies. In their experiments the players whose iterated-dominance strategies were not dominant strategies did not systematically choose the former. However, the majority of players with weakly dominant strategies did select them. These results have been replicated in other similar experiments and in experiments involving more iterations. The problem appears to be that while subjects in experiments...
are able to perform a number of steps of iterated reasoning for themselves they are less willing to believe that other players are able to do the same (see Camerer, 2003: 200–9 for a summary of these experimental results). Nevertheless this method can still be a useful and intuitively plausible way of resolving games like battle of the Bismarck Sea.

2.3 Nash equilibrium

In a Nash equilibrium the players in a game choose strategies that are best responses to each other. However, no player’s Nash-equilibrium strategy, or more simply their Nash strategy, is necessarily a best response to any of the other strategies of the other players. Nevertheless, if all the players in a game are playing their Nash strategies none of the players has an incentive to do anything else. In every dominant strategy and iterated-dominance equilibrium the players’ strategies are also best responses to each other. Therefore every dominant strategy and iterated-dominance equilibrium must also be a Nash equilibrium. But not every Nash equilibrium is also a dominant strategy equilibrium or even an iterated-dominance equilibrium. Consequently there are games that have no dominant strategy or iterated-dominance equilibrium but do have a Nash equilibrium. However, some games have no equilibrium at all in pure strategies, as you will see.

Nash equilibrium

- A combination of players’ strategies that are best responses to each other.

To help you to understand the concept of a Nash equilibrium we are going to look at a number of examples. The first of these is called computer wars 1. This game is represented in pay-off Matrix 2.12. The players in computer wars 1 are two computer companies that are simultaneously planning newspaper advertising campaigns. They plan their campaigns in secret and run them simultaneously. A promotional offer is an integral part of any campaign they run. Both companies choose between offering a lower price, a free printer or an extended guarantee. The pay-offs in Matrix 2.12 represent expected profits. The pay-offs show that whatever Chip offers, it is in Pin’s best interests to make a different offer unless Chip offers the extended guarantee, in which case Pin should match Chip’s offer. Chip always wants to match Tell’s offer.
To find the Nash equilibrium of computer wars 1 we need to identify each player’s best response to each of the other’s strategies. We could start by identifying Pin’s best responses to each of Chip’s three possible strategies. Then we could identify Chip’s best responses to each of Pin’s three possible strategies. If any two of the strategies we identify are best responses to each other then we will have found a strategy combination that constitutes a Nash equilibrium. This sounds complicated but once the best responses of each player are found it is straightforward to identify a Nash equilibrium if one exists. The trick then is to identify both players’ best response strategies. The way we will do this here is by underlining the pay-offs corresponding to each player’s best response to each of the strategies of the other. We can call these pay-offs their ‘best response’ pay-offs. If we follow this procedure for each player then any cell where both pay-offs are underlined will identify a Nash equilibrium. By the way, underlining isn’t a requirement. You can identify the pay-offs corresponding to a player’s optimal strategies in any way you choose e.g. by a * or a circle – indeed whatever takes your fancy. But underlining works just as well as anything else.

To see how the underlining method works let’s start by considering Pin’s position. If Chip chooses lower price then Pin’s best response is to offer a free printer; by choosing free printer his pay-off is 4 whereas he only gets 3 by choosing an extended guarantee and 0 by choosing to lower price. So in Matrix 2.12.1 I have underlined Pin’s pay-off of 4 in the first cell of the middle row. If Chip chooses free printer then Pin’s best response is to lower price so I have underlined his pay-off of 4 in the first row of the second column. If Chip chooses to offer an extended guarantee then Pin’s best response is to also offer an extended guarantee and so I have underlined Pin’s pay-off of 6 in the last row of the third column; he gets a pay-off of 6 by matching Chip and only 5 otherwise.

Matrix 2.12  Computer wars 1

<table>
<thead>
<tr>
<th>Pin Ltd</th>
<th>Chip Inc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lower price</td>
</tr>
<tr>
<td>lower price</td>
<td>0, 4</td>
</tr>
<tr>
<td>free printer</td>
<td>4, 0</td>
</tr>
<tr>
<td>extended guarantee</td>
<td>3, 5</td>
</tr>
</tbody>
</table>
**Matrix 2.12.1 Pin’s best responses to Chip**

<table>
<thead>
<tr>
<th>Pin Ltd</th>
<th>Chip Inc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lower price</td>
</tr>
<tr>
<td>lower price</td>
<td>0, 4</td>
</tr>
<tr>
<td>free printer</td>
<td>4, 0</td>
</tr>
<tr>
<td>extended guarantee</td>
<td>3, 5</td>
</tr>
</tbody>
</table>

Following the same procedure for Chip leads to the pattern of underlining in Matrix 2.12.2. If Pin chooses lower price then Chip’s best response is also to lower price and so I have underlined Chip’s pay-off of 4 in the first cell of the top row. If Pin offers a free printer then Chip’s best response is to also offer a free printer and so I have underlined Chip’s pay-off of 4 in the middle row of the second column. If Pin offers the extended guarantee then Chip’s best response is again to match Pin’s offer as by doing this Chip’s pay-off is 6 which is higher than the pay-off of 3 that results if he chooses either of the alternatives.

**Matrix 2.12.2 Chip’s best responses to Pin**

<table>
<thead>
<tr>
<th>Pin Ltd</th>
<th>Chip Inc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lower price</td>
</tr>
<tr>
<td>lower price</td>
<td>0, 4</td>
</tr>
<tr>
<td>free printer</td>
<td>4, 0</td>
</tr>
<tr>
<td>extended guarantee</td>
<td>3, 5</td>
</tr>
</tbody>
</table>

In Matrix 2.12.3 both players’ best response pay-offs are underlined. The only cell with two underlinings is the third cell of the bottom row which is highlighted. The pay-off pair (6, 6) is the outcome if both players choose the extended guarantee. The double underlining means that choosing the extended guarantee is Pin’s best response if Chip chooses the extended guarantee and choosing the extended guarantee is also a best response for Chip if Pin chooses the extended guarantee. Thus each player’s strategy is a best response to the other’s implying that {extended guarantee, extended guarantee} is the Nash equilibrium of computer wars 1.
To see that every dominant strategy equilibrium is also a Nash equilibrium we can look at the game in Matrix 2.13. Do you recognise this game? It is the same as the pub managers’ game you saw represented in Matrix 2.1. You have already seen that the dominant-strategy equilibrium of this game is {special offer, special offer}. In Matrix 2.13 both managers’ best response pay-offs are underlined. For the manager of the Queen’s Head special offer is a best response to both special offer and no offers. I have therefore underlined the Queen’s Head’s pay-offs of 10 and 18 in the top row of the matrix. Special offer is also a best response for the manager of the King’s Head to either special offer or no offer by the Queen’s Head. I have therefore underlined the King’s Head manager’s pay-offs of 14 and 20 in the first column of the matrix. The two underlinings in the first cell of the top row show that special offer is a best response to special offer and that {special offer, special offer} is a Nash equilibrium as well as a dominant-strategy equilibrium.

Matrix 2.13  Pub managers game

<table>
<thead>
<tr>
<th></th>
<th>special offer</th>
<th>no offer</th>
</tr>
</thead>
<tbody>
<tr>
<td>special offer</td>
<td>10, 14</td>
<td>18, 6</td>
</tr>
<tr>
<td>no offer</td>
<td>4, 20</td>
<td>7, 8</td>
</tr>
</tbody>
</table>
and therefore it is appropriate to underline both of the corresponding pay-offs. For Ms Row club is a best response to party but party is a best response to club. As club is a best response to party for Ms Row and party is a best response to club for Mr Column {club, party} is a Nash equilibrium as well as an iterated-dominance equilibrium.

**Matrix 2.14  Friends or enemies 3**

<table>
<thead>
<tr>
<th>Ms Row</th>
<th>Mr Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>party</td>
<td>party</td>
</tr>
<tr>
<td>club</td>
<td>club</td>
</tr>
</tbody>
</table>

Because every dominant-strategy equilibrium and iterated-dominance equilibrium is also a Nash equilibrium it may be simpler, when looking for the equilibrium of the game, to start by looking for the Nash equilibrium. After identifying a Nash equilibrium it is relatively straightforward to check whether the Nash equilibrium is also a dominant strategy equilibrium or an iterated-dominance equilibrium. Look again at the Pub managers game where the Nash equilibrium is also a dominant-strategy equilibrium. The underlinings in pay-off Matrix 2.13 corresponding to the Queen’s Head’s best response pay-offs are all in the same (the top) row. Similarly, the underlinings identifying the King’s Head’s best response pay-offs are all in the same (the first) column. This shows in a visual way that each player has a dominant strategy and that the Nash equilibrium is also a dominant-strategy equilibrium. In friends or enemies 3 in Matrix 2.14 the situation is a bit different. Two of the three underlinings identifying Mr Column’s best response pay-offs are in the same column (the first) but each of the underlinings identifying Ms Row’s best response pay-offs are in different rows. This shows that Mr Column but not Ms Row has a weakly dominant strategy and therefore the Nash equilibrium in this case is also a weak iterated-dominance equilibrium.

**Exercise 2.4**

In the version of computer wars in Matrix 2.15 Chip and Pin can only choose between lower price and free printer. Pin has secured a large consignment of cut price printers and free printer is now a dominant strategy for Pin but Chip still doesn’t have a dominant strategy. What is the Nash equilibrium of computer wars 2? Is the Nash equilibrium also an iterated-dominance equilibrium and if so is it strong or weak?
2.3.1 Some formal definitions

To give a formal definition of a dominant strategy for player A in a two-person game played with player B it is convenient to define the following:

(i) \( P(A_i, B_i) \) is player A's pay-off from choosing strategy \( A_i \) when player B chooses strategy \( B_i \).

(ii) \( P(A_{-i}, B_i) \) is player A's pay-off from choosing some strategy other than \( A_i \) when player B chooses strategy \( B_i \).

(iii) \( P(A_i, B_{-i}) \) is player A's pay-off from choosing \( A_i \) when player B chooses some strategy other than \( B_i \).
With the above definitions $A_i$ is a strictly dominant strategy for player $A$ if for all the possible alternative strategies $A_{-i}$ and $B_{-i}$:

$$P(A_i, B_i) > P(A_{-i}, B_i) \text{ and } P(A_i, B_{-i}) > P(A_{-i}, B_{-i})$$  \hspace{2cm} \text{condition (2.1)}

Condition (2.1) implies that all the $A_{-i}$ are dominated strategies. If either of the strict inequalities are equalities then $A_i$ is only a weakly dominant strategy.

A dominant-strategy equilibrium is a combination of strategies where every strategy of every player is a dominant strategy. Thus if we define the following in relation to player $B$'s strategies:

(iv) $P(B_i, A_i)$ is player $B$'s pay-off from choosing strategy $B_i$ when player $A$ chooses strategy $A_i$.

(v) $P(B_{-i}, A_i)$ is player $B$'s pay-off from choosing some strategy other than $B_i$ when player $A$ chooses strategy $A_i$.

(vi) $P(B_i, A_{-i})$ is player $B$'s pay-off from choosing $B_i$ when player $A$ chooses some strategy other than $A_i$.

Then $B_i$ is a strictly dominant strategy for $B$ if for all the possible alternatives $B_{-i}$ and $A_{-i}$:

$$P(B_i, A_i) > P(B_{-i}, A_i) \text{ and } P(B_i, A_{-i}) > P(B_{-i}, A_{-i})$$  \hspace{2cm} \text{condition (2.2)}

and if both conditions (2.1) and (2.2) are satisfied then $\{A_i, B_i\}$ is a strong dominant strategy equilibrium. If either of the inequalities in conditions (2.1) and (2.2) are equalities then $\{A_i, B_i\}$ is only a weak dominant strategy equilibrium.

Using definitions (i)–(ii) and (iv)–(v) above $A_i$ and $B_i$ will constitute a Nash equilibrium if:

$$P(A_i, B_i) > P(A_{-i}, B_i) \text{ and } P(B_i, A_i) > P(B_{-i}, A_{-i})$$  \hspace{2cm} \text{condition (2.3)}

If either of the inequalities in condition (2.3) is an equality then the Nash equilibrium is weak, otherwise it is strong.

Note that definitions (iii) and (vi) are not needed to define a Nash equilibrium. Now compare conditions (2.1) and (2.2) with condition (2.3). The first inequality in condition (2.3) is the same as the first inequality in condition (2.1) and the second inequality in condition (2.3) is the same as the first inequality in condition (2.2). Hence if conditions (2.1) and (2.2) are satisfied so is condition (2.3). This means that condition (2.3) is a necessary but not a sufficient condition for $A_i$ and $B_i$ to constitute a dominant-strategy equilibrium and therefore every dominant-strategy equilibrium must also be a Nash equilibrium. Condition (2.3) is also a necessary condition for $A_i$ and $B_i$ to constitute an iterated-dominance equilibrium if for at least one of the players the relevant inequality also holds with respect to all of the other player's non-dominated strategies.
A number of well-defined two-person simultaneous-move games have been used to generate general inferences about a range of strategic situations. These games have been around for many years and are widely used as illustrative examples. They include games of assurance, battle of the sexes, chicken and the war of attrition. They generally have multiple or problematic Nash equilibria. Some examples are analysed here. The whole of the next chapter is devoted to the prisoners’ dilemma, probably the most famous strategic game of all.

2.4.1 Ranked coordination: coordination with assurance

In coordination games the players have an incentive to coordinate their strategies in order to secure mutually beneficial outcomes or avoid mutually harmful ones. This will be more difficult in games with more than one Nash equilibrium. Have a look at the matching moves game represented in Matrix 2.17. The players in this game are managers of two firms who want to coordinate their price strategies. Their pay-offs represent their profits. There are two Nash equilibria in this game – can you identify them?

Matrix 2.17  Matching moves

<table>
<thead>
<tr>
<th>Firm X</th>
<th>Firm Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>raise price</td>
</tr>
<tr>
<td>raise price</td>
<td>5, 5</td>
</tr>
<tr>
<td>lower price</td>
<td>2, 1</td>
</tr>
</tbody>
</table>

In matching moves the Nash equilibria are \{raise price, raise price\} and \{lower price, lower price\}. Do you think one of them is more likely to be the outcome of this game than the other? Well, in the \{raise price, raise price\} equilibrium both firms’ pay-offs are higher than in the \{lower price, lower price\} equilibrium. Therefore both players prefer the former and for this reason it seems intuitively more likely to be the outcome of the game. The problem with the \{lower price, lower price\} equilibrium is that both players can benefit by switching to the \{raise price, raise price\} outcome. An outcome where at least one of the players can benefit if one or both does something else, without worsening the position of the other is called a Pareto inefficient outcome. \{lower price, lower price\} is a Pareto inefficient outcome as both players can benefit by changing their strategy to raise price. The \{raise price, raise price\} equilibrium on the other hand is
Pareto efficient as neither player could benefit by switching strategies without lowering the pay-off of the other. Since both players are advantaged by switching their strategies from lower price to raise price the \{raise price, raise price\} equilibrium is said to Pareto dominate the \{lower price, lower price\} alternative.

The firms’ shared interests in securing the higher ranked Nash equilibrium \{raise price, raise price\} provides them with an element of assurance when they are choosing their strategies. Because \{raise price, raise price\} is advantageous to both of them it appears to be the more compelling of the two Nash equilibria. It may be that it acts as a kind of focal point for the players in that it stands out or has prominence and therefore they are able to coordinate their choices around it.8

However, in games with multiple equilibria Pareto domination won’t automatically guarantee coordination. Consider the matching moves game in Matrix 2.17.1 where the pay-offs of Firm Y are a little different from those in Matrix 2.17. In the game in Matrix 2.17.1 the raise price strategy is risky for the manager of Firm Y. If he chooses raise price and for some reason Firm X does not, Firm Y’s pay-off is –100. This is a lot less than he gets if he chooses lower price and Firm X chooses raise price. This added risk for Firm Y makes the Pareto dominant Nash equilibrium \{raise price, raise price\} seem less likely.

<table>
<thead>
<tr>
<th>Firm X</th>
<th>Firm Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>raise price</td>
</tr>
<tr>
<td>raise price</td>
<td>5, 5</td>
</tr>
<tr>
<td>lower price</td>
<td>2, –100</td>
</tr>
</tbody>
</table>

- **Pareto efficiency**: an outcome is Pareto efficient if it is not possible to improve the pay-off of one player without lowering the pay-off of another.
- **Pareto domination**: outcome 1 Pareto dominates or is Pareto superior to outcome 2 if the pay-offs of one or more players is higher and none are lower in outcome 1.
- **Pareto inefficiency**: an outcome is Pareto inefficient if it is Pareto dominated by another outcome.
2.4.2 Weak ranking

In the game represented in Matrix 2.18 the two players are Mr English and Mr French who are driving their horse-drawn carriages towards each other along a track in Victorian England. Mr English has a preference for driving on the left and Mr French has a preference for driving on the right. Because they are in England where more of the people share Mr English’s preferences, Mr English feels more strongly about driving on the left than Mr French does about driving on the right. There are two Nash equilibria in this game – can you identify them?

Matrix 2.18 Which side of the track?

<table>
<thead>
<tr>
<th>Mr English</th>
<th>right</th>
<th>left</th>
</tr>
</thead>
<tbody>
<tr>
<td>right</td>
<td>2, 3</td>
<td>0, 0</td>
</tr>
<tr>
<td>left</td>
<td>1, 1</td>
<td>3, 3</td>
</tr>
</tbody>
</table>

The two Nash equilibria in Which side of the track? are \{left, left\} and \{right, right\}. However, there is less assurance than in matching moves as \{left, left\} only weakly Pareto dominates \{right, right\}; Mr English prefers \{left, left\} but Mr French is indifferent between \{left, left\} and \{right, right\}. In this game Mr English’s preference for \{left, left\} provides some assurance for Mr French that Mr English will choose left. This should perhaps encourage him to choose left himself. Mr English knows this and therefore \{left, left\} still seems the likely outcome even though it only weakly Pareto dominates \{right, right\}.

However, some further doubts about the strength of Pareto domination as a selection criteria are raised by the related experimental evidence.\(^9\) Van Huyck, Battalio and Beil (1990) conducted a series of coordination games with Pareto ranked multiple equilibria. They found that subjects were unlikely to make initial choices that corresponded to the Pareto dominant equilibrium although in some cases players did converge to it after a number of repetitions. This was more likely when fewer players were involved. Cooper, DeJong, Forsythe and Ross (1990) ran experiments where respondents played two player games with a choice of three strategies. Each game had two Pareto ordered Nash equilibria. They found that Pareto dominance was not automatically a selection criteria. Subjects were also less likely to select strategies consistent with the Pareto dominant equilibrium when these strategies were associated with the kind of risk experienced by Firm Y in Matrix 2.17.1. An interpretation of these results suggested by Cooper et al. is that individuals may be uncertain as to the rationality of the other player in the game. In other experiments Cooper, DeJong, Forsythe and Ross (1989) found that subjects were much more likely to select the Pareto dominant equilibrium when one-way pre-play communication was allowed.\(^{10}\)
2.4.3 Coordination without assurance

In coordination games without assurance there are multiple Nash equilibria but none of them Pareto dominates. In the Battle of the sexes game represented in Matrix 2.19 the players want to meet up either at the party or the pub but John has a preference for the pub and Janet has a preference for the party. The worst possible outcome for both of them is that Janet goes to the pub and John goes to the party. But Janet prefers to go to the pub if John is there than to go to the party if he is not and John would rather go to the party if Janet is there than go to the pub without her. You should be able to confirm that the two Nash equilibria are {pub, pub} and {party, party}. The problem for Janet and John is that John prefers the first equilibrium and Janet the second so how can they coordinate on either? There is no obvious answer to this question and in experiments involving battle of the sexes games coordination failure is common (see Camerer, 2003: Chapter 7). One way may be for one of the players to move first by pre-committing to their preferred venue. For example Jane could pre-commit to the party by buying a present for the host of the party. Alternatively John could pre-commit to the pub by joining the pub darts team. Moving first in this game also gives a player a first-mover advantage. For example, if the party didn’t start till 9 pm John could get a head start on Jane by going down to the pub at 8 pm (see Chapter 5, Section 5.1 where a version of this game is considered in which John moves first).

Matrix 2.19  Battle of the sexes

<table>
<thead>
<tr>
<th></th>
<th>pub</th>
<th>party</th>
</tr>
</thead>
<tbody>
<tr>
<td>pub</td>
<td>3,2</td>
<td>1,1</td>
</tr>
<tr>
<td>party</td>
<td>-1,-2</td>
<td>2,3</td>
</tr>
</tbody>
</table>

Battle of the sexes has applications that go beyond gender relations. For example two food manufacturers may prefer to standardise the ingredients of their product in the interests of promoting consumer confidence, but they may have different preferences over which ingredients to use. Alternatively two neighbouring governments may both wish to adopt minimum wage legislation but they are likely to have different preferences about the level at which to set the minimum (the legislation game in Matrix 2.2 simplifies this problem by assuming that the governments can either introduce the legislation or not – they don’t have discretion about how much legislation to introduce).

Chicken is another coordination game without assurance. There are multiple Nash equilibria but each player prefers a different equilibrium outcome. One of
the non-equilibrium outcomes is truly a disaster for both of them but unlike Battle of the sexes the other is preferred by both players to their least preferred Nash equilibrium outcome. In the chicken game in Matrix 2.20, the two players are a couple of ageing boxers who are trying to maintain a media profile by challenging the other to a fight. Neither of them actually wants to fight but by backing down they lose credibility with their fans. If neither of them backs down the fight will go ahead. The two Nash equilibria are \{challenge, back down\} and \{back down, challenge\} but Smith prefers the first of these, in which Jones chooses back down and Jones prefers the second. \{challenge, challenge\} is a disaster for both of them and they both want to avoid this outcome. The other non-equilibrium outcome \{back down, back down\} is preferred by both Jones and Smith to their least preferred Nash equilibrium.

**Matrix 2.20  Chicken 1 (war of attrition)**

<table>
<thead>
<tr>
<th></th>
<th>Jones</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>back down</td>
<td>challenge</td>
</tr>
<tr>
<td>Smith</td>
<td>2, 2</td>
<td>0, 5</td>
</tr>
<tr>
<td></td>
<td>5, 0</td>
<td>-20, -20</td>
</tr>
</tbody>
</table>

In chicken games it is not clear how the players will coordinate their strategies. One possibility considered in Chapter 6 is that the players choose their strategies according to some predetermined probability distribution such as fight with probability \(\frac{1}{4}\) and back down with probability \(\frac{3}{4}\). If players in a game choose their strategies in this way they are using mixed or randomisation strategies. Mixed strategies may have more appeal for players in games that are played over time or repeated. In a one-off game of Chicken 1, if both players choose challenge the -20 pay-off for each player is irretrievable. But in a repeated version of the game, in which both players are randomising between back down and challenge, a -20 pay-off could be recouped by a series of 5s or 2s. A game of chicken played over time is a war of attrition. If chicken 1 is played as a war of attrition then both players start with challenge which gives them a negative pay-off over time and the game continues until one of them chooses back down. The player who maintains the challenge for longest wins the war.

Chicken, especially when played as a war of attrition, has many applications.\(^{13}\) It is possible to conceive of arms races as games of chicken and in 2003 the BBC and the British Government were accused of playing a game of chicken during the Hutton Inquiry because neither side was prepared to shift its position and admit making an error (*Guardian*, 19 August 2003: 4–5).\(^{14}\)
2.4.4 Games of pure conflict

You have already seen two games of pure conflict in Chapter 1 (hide-and-seek and the penalty-taking game). Games of pure conflict are games where there is no scope for coordination because there are no mutually beneficial outcomes: there can only be one winner. Many games of pure conflict are constant-sum games. Consider the game represented in Matrix 2.21. This version of the friends or enemies game is a constant-sum game where the constant-sum is zero. It is a game of pure conflict because Ms Row doesn’t care whether she goes to the party or the club, she just wants to avoid Mr Column. Mr Column on the other hand wants to see Ms Column so much that he too doesn’t care whether he goes to the party or the club, he just wants to go where she goes. Does friends or enemies 4 have a Nash equilibrium?

Matrix 2.21 Friends or enemies 4

<table>
<thead>
<tr>
<th></th>
<th>party</th>
<th>club</th>
</tr>
</thead>
<tbody>
<tr>
<td>party</td>
<td>-1, 1</td>
<td>1, -1</td>
</tr>
<tr>
<td>club</td>
<td>1, -1</td>
<td>-1, 1</td>
</tr>
</tbody>
</table>

Matrix 2.21.1 shows the best responses of Ms Row and Mr Column underlined. There is no cell in which both players’ pay-offs are underlined and therefore no Nash equilibrium. There is no Nash equilibrium because there are no strategy pairs where the strategies are best responses to each other. For example, if both players choose party then Ms Row will want to switch to club and if she switches to club then Mr Column will want to switch from party. But if he does Ms Row will want to switch back to party. Every possible strategy combination is like this. One of the players will always want to deviate. Consequently there is no Nash equilibrium in friends or enemies 4, or to be more precise no Nash equilibrium in pure strategies.

Matrix 2.21.1 Friends or enemies 4: both player’s best responses

<table>
<thead>
<tr>
<th></th>
<th>party</th>
<th>club</th>
</tr>
</thead>
<tbody>
<tr>
<td>party</td>
<td>-1, 1</td>
<td>1, -1</td>
</tr>
<tr>
<td>club</td>
<td>1, -1</td>
<td>-1, 1</td>
</tr>
</tbody>
</table>
Similar reasoning applies in many but not all games of pure conflict. The battle of the Bismarck Sea is an exception. It is a game of pure conflict but there is a Nash equilibrium because one of the players has a (weakly) dominant strategy. In games with no Nash equilibrium in pure strategies it is difficult to predict what will happen. This problem is compounded in constant-sum games because unlike battle of the sexes or chicken there is a first-mover disadvantage. In Friends or enemies 4 if Ms Row moves first by going to the party Mr Column will surely follow which will be to Ms Row’s disadvantage. Mr Column is in the same position. If he moves first Ms Row will just as surely avoid him which will be to his disadvantage. When there is a first-mover disadvantage the secret of success is to make your moves unpredictable. One way to do this is to act unsystematically by choosing between strategies in a random way. If a player does this they are choosing a mixed rather than a pure strategy. It turns out that all simultaneous-move two-player games, including constant-sum games, have a Nash equilibrium in mixed strategies (Glicksberg, 1952). Therefore it is a theoretical possibility for the players in a game with no Nash equilibrium in pure strategies to coordinate on a Nash equilibrium in mixed strategies. This possibility is discussed in detail in Chapter 6.

**Exercise 2.6**

Taking a penalty 2 is a variation of taking a penalty 1, the game you saw in Chapter 1. In this version of the game the striker gains more satisfaction if he scores by kicking the ball into the corners of the goal. Does taking a penalty 2 have a Nash equilibrium (in pure strategies)? If not can you explain why? Does either player in this game have a first-over advantage?

**Matrix 2.22 Taking a penalty 2**

<table>
<thead>
<tr>
<th></th>
<th>goalkeeper</th>
</tr>
</thead>
<tbody>
<tr>
<td>left</td>
<td>left</td>
</tr>
<tr>
<td></td>
<td>0, 1</td>
</tr>
<tr>
<td></td>
<td>2, 0</td>
</tr>
<tr>
<td></td>
<td>2, 0</td>
</tr>
<tr>
<td>middle</td>
<td>1, 0</td>
</tr>
<tr>
<td></td>
<td>0, 1</td>
</tr>
<tr>
<td></td>
<td>1, 0</td>
</tr>
<tr>
<td>right</td>
<td>2, 0</td>
</tr>
<tr>
<td></td>
<td>2, 0</td>
</tr>
<tr>
<td></td>
<td>0, 1</td>
</tr>
</tbody>
</table>
In this chapter a number of simultaneous- or hidden-move games were analysed in detail. You have seen that the analysis of these kinds of games focuses on strategies that are best responses to each other and therefore constitute an equilibrium. Three equilibrium concepts for static games were defined; dominant-strategy equilibrium, iterated-dominance equilibrium and Nash equilibrium (in pure strategies). You have learned how to derive each of these in two-person simultaneous-move games.

In a Nash equilibrium the players’ strategies are best responses to each other. In a dominant-strategy equilibrium the players’ strategies are best responses to all of the other players’ strategies. In an iterated-dominance equilibrium the players’ strategies are best responses not only to each other but also, for at least one of the players, to some of the other strategies of the other player. Because the conditions that need to be satisfied for a Nash equilibrium are necessary but not sufficient conditions for a dominant strategy and iterated-dominance equilibrium, every dominant-strategy and iterated-dominance equilibrium is also a Nash equilibrium. But not every Nash equilibrium is also a dominant-strategy or an iterated-dominance equilibrium. It may therefore be simpler when searching for the theoretical outcome of a game to start by looking for a Nash equilibrium and then, if one is found, check whether it is either a dominant-strategy or iterated-dominance equilibrium. If Nash equilibrium strategies are also dominant strategies then we can be more confident about predicting the Nash equilibrium as the outcome of the game.

A straightforward way of finding a Nash equilibrium is to underline or otherwise identify in the game’s pay-off matrix each player’s ‘best response’ pay-offs. These are the pay-offs that correspond to their best responses to each of the other players’ strategies. After following this procedure you can look for cells in the pay-off matrix where both players’ pay-offs are identified as best response pay-offs. The strategies corresponding to these pay-offs will be best responses to each other and will constitute a Nash equilibrium.

In Section 2.4 some classic games including chicken and battle of the sexes were analysed. You saw that some simultaneous two-player games have more than one Nash equilibrium and others have none. In games with assurance there are multiple equilibria but one of the Nash equilibria seems more plausible by virtue of Pareto dominance. However, in games like chicken it is difficult to predict how the players will coordinate their strategy choices. In games of pure conflict there may be no Nash equilibrium in pure strategies and one possibility is that players will try to create doubt in their opponent’s mind by choosing mixed strategies.
The Nash equilibrium is an important concept that is used extensively in game theory. It does, however, have some limitations. First of all, as you have already seen, some games have multiple Nash equilibria and some have no Nash equilibria in pure strategies. Secondly, as you will see in the following chapters, the problem of multiple Nash equilibria gets worse as games become more complicated. This has led game theorists to refine the concept of a Nash equilibrium when moves are sequential and information is not perfect. Sequential move games are analysed in Chapter 4. In these games refinement of Nash equilibrium leads to the idea of a subgame perfect Nash equilibrium. In games with imperfect information the process of refinement leads to the concept of a Bayesian Nash equilibrium (see Chapter 7). Last but not least a long line of academics have raised objections to the underlying assumptions of Nash equilibrium such as rationality and common knowledge.

2.1
Whatever Bart does Art is always better off choosing invest; Art gets at most 3 by not investing and either 9 or 5 by investing. Whatever Art does Bart is also better off choosing invest. Consequently the dominant-strategy equilibrium is {invest, invest}

2.2
The dominant-strategy equilibrium is {party, party}. Both players prefer to go to the party whatever the other player does.

2.3
A wide range of correct answers is possible. Pay-off Matrix 2.7.1 shows one possibility. In Matrix 2.7.1 the pay-offs make no challenge a strongly dominated strategy for the challenger and the strong iterated-dominance equilibrium is {challenge, resign}.

Matrix 2.7.1 Political ambition with a weak incumbent MP

<table>
<thead>
<tr>
<th>Challenger</th>
<th>Incumbent MP</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>stand</td>
<td>resign</td>
</tr>
<tr>
<td>no challenge</td>
<td>5, 10</td>
<td>0, 1</td>
</tr>
<tr>
<td>challenge</td>
<td>10, -15</td>
<td>15, 1</td>
</tr>
</tbody>
</table>
2.4
The Nash equilibrium of computer wars 2 can be found in three steps:

Step 1: Underline or otherwise indicate the pay-offs corresponding to Pin's best responses to each of Chip's strategies as shown in Matrix 2.15.1.

**Matrix 2.15.1 Pin's best responses**

<table>
<thead>
<tr>
<th>Pin Ltd</th>
<th>Chip Inc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>lower price</td>
</tr>
<tr>
<td>lower price</td>
<td>1, 6</td>
</tr>
<tr>
<td>free printer</td>
<td>6, 2</td>
</tr>
</tbody>
</table>

Step 2: Underline the pay-offs corresponding to Chip's best responses to each of Pin's two strategies as shown in Matrix 2.15.2.

**Matrix 2.15.2 Chip's best responses**

<table>
<thead>
<tr>
<th>Pin Ltd</th>
<th>Chip Inc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>lower price</td>
</tr>
<tr>
<td>lower price</td>
<td>1, 6</td>
</tr>
<tr>
<td>free printer</td>
<td>6, 2</td>
</tr>
</tbody>
</table>

Step 3: Combine the two matrices and check to see if a cell in the pay-off matrix has two underlinings as shown in Matrix 2.15.3.

**Matrix 2.15.3 Both firms' best responses**

<table>
<thead>
<tr>
<th>Pin Ltd</th>
<th>Chip Inc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>lower price</td>
</tr>
<tr>
<td>lower price</td>
<td>1, 6</td>
</tr>
<tr>
<td>free printer</td>
<td>6, 2</td>
</tr>
</tbody>
</table>

There are two underlinings in the (highlighted) cell in the bottom row of the second column. This implies that offering the free printer is a best response by both players if the other also offers a free printer. \{free printer, free printer\} is therefore the Nash equilibrium of the game. This strategy combination is also a strong iterated-dominance equilibrium found by deleting Dime's strongly dominated strategy of lower price.
2.5
Pay-off Matrix 2.16.1 shows the best responses of both players underlined. The Nash equilibrium is \{lower price, free printer\}. This strategy combination is also a strong iterated-dominance equilibrium found by deleting Chip’s strongly dominated strategy extended guarantee which makes free printer strongly dominated for Pin. Deleting Pin’s strategy of free printer makes lower price strongly dominated for Tell.

Matrix 2.16.1

<table>
<thead>
<tr>
<th></th>
<th>Chip Inc</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lower price</td>
<td>free printer</td>
</tr>
<tr>
<td>Pin Ltd</td>
<td>lower price</td>
<td>1, 0</td>
</tr>
<tr>
<td></td>
<td>free printer</td>
<td>0, 3</td>
</tr>
</tbody>
</table>

2.6
There is no Nash equilibrium in pure strategies in taking a penalty 2. Even though the pay-offs do not sum to a constant the game is still one of pure conflict and neither player has a dominant strategy; if the striker scores the goalkeeper effectively loses and vice versa. Neither player has a first-mover advantage, there is a first-mover disadvantage.

Problems

1. Identify the Nash equilibria of the up-down, left-right game represented in Matrix 2.23. Is there more than one Nash equilibrium? If so are all the Nash equilibria also iterated-dominance equilibria? Are the iterated-dominance equilibria that exist strong or weak?

Matrix 2.23  The up-down, left-right game

<table>
<thead>
<tr>
<th></th>
<th>player A</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>player B</td>
<td>up</td>
<td>down</td>
</tr>
<tr>
<td></td>
<td>left left</td>
<td>left right</td>
</tr>
<tr>
<td></td>
<td>2, 0</td>
<td>4, 2</td>
</tr>
<tr>
<td></td>
<td>2, 0</td>
<td>1, 1</td>
</tr>
</tbody>
</table>
2 Identify the Nash equilibria in the chicken game in Matrix 2.24. What kind of situation do you think is being modelled in chicken 2?

**Matrix 2.24  Chicken 2**

<table>
<thead>
<tr>
<th>Jessie</th>
<th>Rosie</th>
</tr>
</thead>
<tbody>
<tr>
<td>stay</td>
<td>-10, -10</td>
</tr>
<tr>
<td>swerve</td>
<td>-1, 2</td>
</tr>
</tbody>
</table>

3 In the stag hunt game in Matrix 2.25 each player chooses between hunting a stag (which will only be successful if both players join in) and shooting a hare (which doesn’t require the help of anyone else). There are two Nash equilibria in this game – which do you think is more likely?

**Matrix 2.25  Stag hunt**

<table>
<thead>
<tr>
<th>player 1</th>
<th>player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>stag</td>
<td>5, 5</td>
</tr>
<tr>
<td>hare</td>
<td>1, 0</td>
</tr>
</tbody>
</table>

1 Explain what is implied by a Nash equilibrium (in pure strategies) in a simultaneous-move game.

2 Why is every dominant-strategy equilibrium also a Nash equilibrium?

3 In what kinds of circumstances might the Nash equilibrium concept be of limited use in predicting the outcome of a game?

4 How do you think that games of pure conflict like penalty taking are resolved in practice?
1 There are 3 Nash equilibria as shown in Matrix 2.23.1. They are: \{down, left left\}, \{up, right left\} and \{down, right right\}. The last of these is a weak iterated-dominance equilibrium found initially by deleting B’s strongly dominated strategy left right. And then left left and right left.

Matrix 2.23.1 The up-down, left-right game

<table>
<thead>
<tr>
<th></th>
<th>left left</th>
<th>left right</th>
<th>right left</th>
<th>right right</th>
</tr>
</thead>
<tbody>
<tr>
<td>up</td>
<td>2, 0</td>
<td>2, 0</td>
<td>3, 3</td>
<td>3, 3</td>
</tr>
<tr>
<td>down</td>
<td>4, 2</td>
<td>1, 1</td>
<td>1, 1</td>
<td>4, 2</td>
</tr>
</tbody>
</table>

2 In chicken 2, the two Nash equilibria are \{stay, swerve\} and \{swerve, stay\} but Jessie prefers the first of these and Rosie prefers the second. \{stay, stay\} is a disaster for both players and both players prefer \{swerve, swerve\} to the other’s preferred Nash equilibrium.

This version of chicken represents a classic case in which the two players are playing a game of nerves by driving towards each other or towards a cliff edge or some variation on this theme. The player who swerves first loses face and the player who stays on course the longest wins the glory. But if neither swerves the consequences are disastrous. The game is widely associated with the classic 1955 film Rebel Without a Cause starring James Dean in which the main characters play a variant of this chicken game.

3 The two Nash equilibria are \{stag, stag\} and \{hare, hare\} but the first Nash equilibrium Pareto dominates the second. The rationale is as follows: the stag is bigger and the group is small enough so that a share in the stag is preferred to the whole hare. The situation where both players join in the stag hunt is therefore Pareto superior to the situation where both shoot their own hare.
A series of games like political ambition are analysed in Gates and Humes (1997: Chapter 3).

See Haywood (1954) for an analytical discussion or www.combinedfleet.com/bismksea.htm for more general information.


Deleting weakly dominated strategies may also result in the deletion of a Nash equilibrium.

All references to Nash equilibria in this chapter are to pure strategy Nash equilibria. Remember from Chapter 1 that if a player chooses a pure strategy they choose just one of the alternative strategies that are available to them. If a game doesn’t have an equilibrium in pure strategies it can still have one in mixed strategies as you will see in Chapter 6. Choosing a mixed strategy involves randomising between some or all of the player’s available strategies. A pure strategy can be viewed as a special mixed strategy for which the respective pure strategy is played with probability one and any other strategy with probability zero.

Gibbons (1992: 9) calls this a ‘brute-force approach’ to finding a game’s Nash equilibrium.

A classic coordination game with assurance is Rousseau’s stag hunt. See Problem 3 at the end of this chapter.

Schelling (1960) showed that in many situations where formal theorising doesn’t appear to offer much guidance people are still able to coordinate their actions by focusing independently on some particular feature of the situation.


This kind of communication is sometimes called cheap talk as any commitments made are not binding.

Sometimes the title of this game is changed, for example to the dating game (see Gibbons, 1997: 132) because the original title is considered politically incorrect.

Battle of the sexes is like the friends game but the players have different preferences in relation to the choice of venues.

Another example of a game of chicken is played when two swimmers are swimming in the same lane in a crowded pool. When they are swimming towards each other they face a simultaneous choice of swerving in order to avoid the other or not swerving. If neither swerves there will be an uncomfortable collision. But swerving may set a precedent and is inconvenient. Alternatively, if one of them decides to swim backstroke he can commit to not swerving because he will be unable to see the other.

The Hutton Inquiry investigated the roles of the British Government and the BBC in the death of a senior civil servant who had made claims concerning the contents of a government dossier on weapons of mass destruction in Iraq.

See, for example, Hargreaves Heap and Varoufakis (1997) or Mirowski (2002).
PRISONERS’ DILEMMA

Concepts and techniques

- Prisoners’ dilemma
- Generalised pay-offs
- Pareto efficiency
- Public goods
- Open-access resources
- Binding contracts.

After working through this chapter you will be able to:

- Explain what is implied by a prisoners’ dilemma
- Construct a pay-off matrix for a prisoners’ dilemma game
- Show that the dominant-strategy equilibrium of the prisoners’ dilemma is not Pareto-efficient
- Generalise the pay-offs of the prisoners’ dilemma
- Describe prisoners’ dilemmas in a variety of situations
- Show how prisoners’ dilemma games can be used to analyse problems relating to the provision of public goods and the over-harvesting of open-access resources
- Reflect on some suggestions about ways to resolve prisoners’ dilemmas.
Only one game is examined in this chapter. That game is the prisoners’ dilemma.\textsuperscript{1} The prisoners’ dilemma is a truly classic game in the sense that it has been analysed in countless academic publications and is almost always discussed in introductory reviews of game theory. Its renown has also spread beyond academic circles. This is not surprising as strategic situations that can be characterised as a prisoners’ dilemma are ubiquitous. Applications include oligopoly collusion, international trade and investment, environmental problems, wage inflation and public goods. The prisoners’ dilemma game is interesting not only because of its wide applicability but also because it poses some interesting questions about the underlying assumptions of game theory, specifically in relation to the definition of rationality that the theory employs.\textsuperscript{2} Some of these questions are discussed in this chapter.

This chapter begins with the original application of the prisoners’ dilemma from which the name of the game is derived.\textsuperscript{3} The dilemma is then generalised and applied in a range of contexts in Sections 3.2 to 3.4. Some related policy questions that arise in connection with public goods and the free-rider effect are discussed in Sections 3.5 to 3.6 and a macroeconomic application is discussed in Section 3.7. Some questions raised by the dilemma are discussed in Section 3.8.

### 3.1 Original prisoners’ dilemma game

In the original prisoners’ dilemma two suspects are being interviewed by the police in relation to a major crime. They are being interviewed in separate cells and neither knows how the other’s interview is progressing. The moves of the game are therefore hidden and it is appropriate to model the situation as a simultaneous-move game (even if the prisoners are not actually being interviewed exactly at the same time). An implicit assumption of the game is that the prisoners did in fact commit the crime that they are being questioned about. The suspects can either confess to the crime or deny their involvement in it. If neither prisoner confesses the police are not able to convict either for the major crime but are able to secure a conviction against both of them in relation to a lesser crime. However, if just one of them confesses to the major crime they can both be convicted. The dilemma for the prisoners is that if one of them confesses but the other does not the one who confesses receives a
much shorter sentence than the other (his reward for acting as an informer or ‘grass’).

Take a look at the prisoners’ dilemma represented by the pay-off matrix in Matrix 3.1. In Matrix 3.1 the pay-offs represent the prison sentences that the suspects face as a result of their actions. In this prisoners’ dilemma if one suspect confesses and the other denies the confessor is released while the other receives a ten-year sentence. If neither suspect confesses they both receive short one-year sentences. If both confess they both spend five years in prison. What do you think will be the outcome of this game?

Matrix. 3.1 Prisoners’ dilemma

<table>
<thead>
<tr>
<th></th>
<th>prisoner 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>prisoner 1</td>
<td></td>
</tr>
<tr>
<td>deny</td>
<td>-1, -1</td>
</tr>
<tr>
<td>confess</td>
<td>0, -10</td>
</tr>
</tbody>
</table>

If you apply the methodology of Chapter 2 you will see that game theory makes a clear prediction about the game’s outcome since it has a dominant-strategy equilibrium. You can see this by looking at the pay-off matrix in Matrix 3.1.1 where the prisoners’ pay-offs corresponding to their best responses are underlined. For both prisoners confess is the best response to either deny or confess by the other implying that each player's dominant strategy is to confess. The dominant-strategy equilibrium is therefore {confess, confess} and the game theoretic prediction is that faced with these strategy choices and pay-offs both prisoners will confess.

Matrix. 3.1.1 Dominant-strategy equilibrium of the prisoners’ dilemma

<table>
<thead>
<tr>
<th></th>
<th>prisoner 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>prisoner 1</td>
<td></td>
</tr>
<tr>
<td>deny</td>
<td>-1, -1</td>
</tr>
<tr>
<td>confess</td>
<td>0, -10</td>
</tr>
</tbody>
</table>

The dilemma for the players is that they could both have higher pay-offs if they both denied. Since the {confess, confess} equilibrium is Pareto-dominated by {deny, deny} it is not Pareto-efficient. Both prisoners can work out that {confess, confess} is not an efficient outcome (as can the police) but the rational,
self-interested dominant strategy is clearly to confess. This paradoxical result is not resolved by a pre-negotiated agreement to deny as once the police start to question the prisoners they each have an incentive to break the agreement. This will still be true even if they believe that the other will stick to the agreement. The dilemma for the prisoners is that by acting rationally, that is by choosing their strategies to maximise their pay-offs, they are worse off than if they had acted in some other presumably ‘irrational’ way. This is clearly a perverse result. It implies that the players could do better by acting altruistically or even randomly than by acting in their own self-interests. But with the pay-offs as they are in Matrix 3.1 the players will only deny if a prior agreement to deny is somehow enforceable.

Making a prior agreement to deny would clearly be in the prisoners’ joint interests and we could call this jointly rational behaviour because it would make sense if the players were trying to maximise their total rather than their individual pay-offs. But the logic of game theory assumes that individual players choose their strategies to maximise their individual not their joint pay-offs and with this assumption it is not clear how such an agreement could be enforced. And unless the agreement to deny is enforced in some way the incentive for both prisoners to confess is so strong that neither can trust the other to keep to any such agreement. One possibility is that an agreement to deny could be enforced by a threat to punish confession after the event (this could involve a third party in prison or outside). If the punishment for confession was very severe denial could become a dominant strategy (see Problem 2 at the end of the chapter). But then the game would no longer be a prisoners’ dilemma suggesting that changing the pay-offs in this way is a circumvention of the problem rather than a solution.

Pareto efficiency

- In a two-player game an outcome is Pareto-efficient if it is not possible to improve one player’s pay-off without at the same time lowering the pay-off of the other.

The prisoners’ dilemma is not restricted to the scenario described above as played out in many crime dramas on TV and in films. It is therefore useful to characterise the problem in a more general way in order to capture the salient features of the game. Matrix 3.2 shows a generalised pay-off matrix for the prisoners’ dilemma game in Matrix 3.1. A game is a prisoners’ dilemma if the preferences of the players over the pay-offs (a), (b), (c) and (d)
are such that (c) is preferred to (a), (a) is preferred to (d) and (d) is preferred to (b) which, as more is assumed to be preferred to less, implies that $c > a > d > b$ as indicated below. In Matrix 3.2, I have underlined the pay-offs that correspond to each player's best responses. As you can see, confess is still a dominant-strategy for both prisoners. The dominant-strategy equilibrium is therefore for both prisoners to confess but as $a > d$ both of them would be better off if they could both deny.

### Matrix 3.2 Generalising the pay-offs in the prisoners’ dilemma

<table>
<thead>
<tr>
<th></th>
<th>prisoner 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>deny</td>
</tr>
<tr>
<td>deny</td>
<td>$a, a$</td>
</tr>
<tr>
<td>confess</td>
<td>$c, b$</td>
</tr>
</tbody>
</table>

$c > a > d > b$

Any game with the pay-off structure of Matrix 3.2 is a prisoners’ dilemma. The players do not have to be prisoners and their strategy choices will rarely be between outright denial and confession. To encompass all these different possibilities the deny strategy is generally referred to as the cooperative strategy and the confess strategy is referred to as the defect strategy. By cooperating the players can achieve a mutually beneficial outcome. Defection, on the other hand, can be mutually harmful. If the prisoners in the original example both denied they would be cooperating or colluding with each other in order to achieve a shorter sentence. By confessing a prisoner is defecting from the cooperative strategy. A prisoners’ dilemma with these generalised strategies and generalised pay-offs is shown in Matrix 3.3. The dominant-strategy equilibrium of the game in Matrix 3.3 is {defect, defect} even though both players would be better off if they could both cooperate.

### Matrix 3.3 Generalising the strategies as well as the pay-offs in the prisoners’ dilemma

<table>
<thead>
<tr>
<th></th>
<th>player column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cooperate (with row)</td>
</tr>
<tr>
<td>cooperate (with column)</td>
<td>$a, a$</td>
</tr>
<tr>
<td>defect</td>
<td>$c, b$</td>
</tr>
</tbody>
</table>

$c > a > d > b$
Many of the strategic issues facing managers of firms in oligopoly markets can be modelled using game theory and one of the most cited examples is a prisoners’ dilemma. The application of the prisoners’ dilemma to oligopoly theory refers to the problem for firms of sustaining cartels or more implicit collusion over prices, output or other competitive weapons such as spending on advertising. These kinds of agreements stifle competition and are not usually in the interests of consumers but they are desirable from the firms’ perspective because they can raise profits. Consider the strategic situation described by the pay-off matrix in Matrix 3.4.

In the game of oligopoly collusion represented in Matrix 3.4 Ash and Birch are the only two firms producing wood flooring in Jesmania. The wood flooring market is therefore an oligopoly or more precisely a duopoly. Ash and Birch can raise their profits by colluding to maintain a high market price. If they do this they each make profits of 7 billion units of Jesmanian money. The dilemma for the firms is that if one of them cheats on the agreement by lowering their price the cheat’s profits rise to 10 billion while the other loses custom (to the cheat) and profits fall to 3 billion. If both firms cheat by cutting price neither firm gains customers from the other and the profits of both firms fall to 5 billion.

Matrix 3.4  Oligopoly collusion

<table>
<thead>
<tr>
<th></th>
<th>collude</th>
<th>cheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>collude</td>
<td>7, 7</td>
<td>3, 10</td>
</tr>
<tr>
<td>cheat</td>
<td>10, 3</td>
<td>5, 5</td>
</tr>
</tbody>
</table>

With the firms’ pay-offs as depicted in Matrix 3.4 do you think that collusion between the firms is likely to be sustained? Game theory suggests that the answer to this question is no. You should be able to see this by working out that cheating is a dominant strategy for both firms and therefore the game
theoretic prediction is that both firms will cheat. This is a prisoners’ dilemma for the firms as they could both make higher profits by colluding. Unfortunately the individual incentives for the firms to cheat are too strong. This prediction can be generalised. It implies that whenever there are strong individual incentives to cheat oligopolistic collusion will be difficult to sustain.10 A real-world example of a collusive agreement breaking down is provided by Sotheby’s and Christie’s. These two international auction houses operated a price-fixing cartel for most of the 1990s until early 2000 in order to reduce the competition between them. The cartel broke down when Christie’s blew the whistle on the cartel and handed over evidence to the European Commission. Christie’s escaped without a fine as a reward for ‘confessing’ while Sotheby’s were fined nearly £13 million.11

However, the incentive to cheat or defect from a collusive agreement won’t always be as strong as it is in Matrix 3.4. If the market share of one firm is considerably larger than that of the other or others then the incentive of the larger firm to cheat may be weakened. Consider what happens if Birch is very large relative to Ash. In this case the pay-off matrix for the game could look like the one in Matrix 3.4.1. In this asymmetric oligopoly game cheating is no longer a dominant strategy for Birch. Birch is so large relative to the market as a whole that breaking the collusive agreement has a negative effect on its own as well as Ash’s profits.12 This is true whether Ash also breaks the agreement or not. Ash’s situation hasn’t changed so cheating is still a dominant strategy for the smaller firm. The dominant strategy equilibrium of this asymmetric game is for Ash to cheat and Birch to collude. This equilibrium outcome is Pareto-efficient as neither firm can improve their pay-off without worsening the position of the other. Consequently the game is no longer a prisoners’ dilemma.

### Matrix 3.4.1  Asymmetric oligopoly collusion

<table>
<thead>
<tr>
<th></th>
<th>collude</th>
<th>cheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>collude</td>
<td>7, 40</td>
<td>3, 18</td>
</tr>
<tr>
<td>cheat</td>
<td>10, 20</td>
<td>5, 10</td>
</tr>
</tbody>
</table>

An example of cooperation being sustained at least partly through the actions of a dominant supplier is OPEC’s ability in the early 1980s to keep oil prices high by restricting output. The OPEC strategy was helped considerably by the willingness of Saudi Arabia, a major player, to withhold production in order that other OPEC members with contrary objectives or in vulnerable political positions (specifically Libya, Iran, Iraq and Nigeria) could exceed their quotas. However, in 1985 Saudi Arabia became unwilling to maintain this position and Saudi production expanded rapidly leading to a virtual collapse of the cartel and a fall in oil prices.
Prisoners’ dilemmas are also found in the arena of international trade. The theory of comparative advantage shows that trade can be mutually beneficial for countries but it is still tempting for governments to try to protect domestic producers from foreign competition by imposing tariffs on imported goods. A tariff helps domestic producers by raising import prices. It will also raise revenue for the government but tariffs will mean higher prices for consumers. A government may be inclined to introduce a tariff if it believes that the benefits of the tariff to domestic industry outweigh the losses to consumers. However, it will still need to take into account the possibility of retaliatory action by other countries. In an extreme case this could escalate into a trade war. Imposing a tariff unilaterally is one thing but if two countries in a trading relationship impose tariffs on each other’s exports, the gains to domestic producers may be outweighed by the revenue losses to domestic exporters. Retaliation of this kind is not uncommon. In 2003 there were fears that a trade war between the USA and Europe would be re-ignited when the USA rejected a final ruling from the World Trade Organisation that its protectionist tariffs on foreign steel were illegal. In retaliation the European Union threatened to impose sanctions on a range of US goods including Harley Davidson motorcycles and Ray-Ban sunglasses.

This kind of scenario is modelled in the international trade game represented in Matrix 3.5. Jesmania and Rosatia are trading partners and each is deciding whether to impose a tariff on imports from the other country or not. If one country imposes a tariff unilaterally then that country makes a significant net gain while the other loses. If both countries impose a tariff then both lose. If neither country imposes a tariff then both make moderate gains from trade. The pay-offs in Matrix 3.5 represent net effects (in billions of euros).

In the international trade game in Matrix 3.5 the dominant-strategy equilibrium is for both countries to impose a tariff even though they would both be better off if neither imposed a tariff. The game as described here is a prisoners’ dilemma for the two countries. The situation might be different if either or
both of the countries were small relative to the market for the traded goods. In this case reduced demand due to a tariff might have little effect on import prices.16 If import prices do not fall the negative effect of a tariff on consumers is more likely to outweigh the positive effects on domestic producers and government revenue. A country in this position has little incentive to unilaterally impose a tariff. This possibility is illustrated in Matrix 3.6 where both countries are assumed to be small and neither has an incentive to impose a tariff.

Matrix 3.6  International trade 2

<table>
<thead>
<tr>
<th></th>
<th>Little Rosatia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no tariff</td>
</tr>
<tr>
<td>Little</td>
<td></td>
</tr>
<tr>
<td>Jesmania</td>
<td>10, 10</td>
</tr>
<tr>
<td>impose tariff</td>
<td>8, -1</td>
</tr>
</tbody>
</table>

International trade 2 is not a prisoners’ dilemma. The dominant-strategy equilibrium of this game is for neither country to impose a tariff, the free trade alternative or mutual cooperation. This result suggests that trade wars are unlikely between small countries that are at the mercy of world markets. Small countries stand to lose more than they gain by imposing tariffs. On the other hand the analysis suggests that trade conflicts will be much more likely to flare up between large countries and large trading blocks like the European Union (EU) and the North American Free Trade Area (NAFTA).

Exercise 3.2

In International trade 3 in Matrix 3.7 one country, Little Rosatia, is assumed to be much smaller relative to the world market than the other, Greater Jesmania. What is the dominant-strategy equilibrium of International trade 3? Can you give an interpretation of this version of the international trade game?

Matrix 3.7  International trade 3

<table>
<thead>
<tr>
<th></th>
<th>Little Rosatia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no tariff</td>
</tr>
<tr>
<td>Greater</td>
<td></td>
</tr>
<tr>
<td>Jesmania</td>
<td>10, 10</td>
</tr>
<tr>
<td>impose tariff</td>
<td>15, -1</td>
</tr>
</tbody>
</table>
The economic definition of a pure public good\(^\dagger\) is a good that is both non-excludable and non-rival in consumption. Non-rivalry means that one person’s consumption does not reduce the supply of the public good to other potential consumers. This implies that supply of the good is non-divisible and the extra cost of supplying it to additional consumers is zero. An example is the security provided by the local police or the protection to sea-going vessels provided by a lighthouse. Non-excludability means that once a good or service becomes available anyone and everyone can use it. That is, no one can be excluded from its consumption. An example is clean air in an unpolluted environment. Most market goods have neither feature. They are private goods meaning that they are both excludable and rival in consumption. If a good is rival then one person’s consumption is at the expense of another’s and the incremental or marginal cost of supplying the good to an additional consumer is therefore positive. If a good is excludable then the owner of the good can exclude anyone and everyone else from using it. In this case the owner of the good is said to have property rights with respect to the good concerned. Some goods are neither purely public nor purely private but lie somewhere in between. In fact many so called public goods are only non-excludable and non-rival up to a point. For instance the security provided by the local police may become rival if there is a riot or major criminal event of some kind. Goods that are neither purely private nor purely public are called mixed goods or impure public goods. Table 3.1 gives some examples. Goods in the top left-hand quadrant are private goods because they are both excludable and rival. Goods in the bottom right-hand quadrant are public goods that are both non-excludable and non-rival. The goods in the other two quadrants are mixed goods. They are either non-excludable but rival or non-rival but excludable.

Table 3.1 Non-rivalry and non-excludability

<table>
<thead>
<tr>
<th>Rival</th>
<th>Non-rival</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure private goods</td>
<td>Mixed goods</td>
</tr>
<tr>
<td>Excludable</td>
<td>Pay per view TV</td>
</tr>
<tr>
<td>Cornflakes</td>
<td>Toll bridges</td>
</tr>
<tr>
<td>Cars</td>
<td>Private roads</td>
</tr>
<tr>
<td>Chocolate</td>
<td></td>
</tr>
<tr>
<td>Mixed goods</td>
<td>Pure public goods</td>
</tr>
<tr>
<td>Non-excludable</td>
<td>National defence</td>
</tr>
<tr>
<td>State education</td>
<td>Lighthouses</td>
</tr>
<tr>
<td>Public health</td>
<td>A clean environment</td>
</tr>
<tr>
<td>Open access resources such as ocean fishing fields, city streets and town parks</td>
<td>Very large national parks</td>
</tr>
</tbody>
</table>
National defence is a public good because it is non-excludable and non-rival. It is non-excludable because once a country has committed to defending itself no citizen can be excluded from the protection it offers; either all are defended or none. It is non-rival because one person’s safety is not secured at the expense of another’s. A lighthouse is a classic public good because no ship can be excluded from the warning it provides and the warning received by one passing ship does not diminish the warning received by the next. Public goods like national defence and the police tend to be provided by governments but not all goods that are provided by governments are pure public goods. Often they are merit goods. Merit goods like state-funded education and the National Health Service in the UK are impure public goods that are funded by governments because they are assumed to have wide-ranging benefits for society.18 Merit goods are usually rival but state provision makes them non-excludable.

Economic theory predicts that the provision of public goods is likely to be problematic. The supply problem stems directly from the non-excludability and non-rivalry characteristics that generate free-rider effects. Free-riders are people who benefit from the provision of a good or service without paying. In the case of a public good the free-rider problem is endemic because no one can be excluded from consumption and one person’s consumption has no effect on another’s. Consequently there are limited private incentives to pay for provision. These free-rider effects can be modelled as a prisoners’ dilemma19 although in most cases more than two players will be involved making the dilemma an n-player game with \( n > 2 \).

Consider a situation where there are two neighbouring communities that both value a threatened natural habitat that has the non-excludable and non-rival characteristics of a public good. The communities are independently considering whether to finance the conservation of the threatened habitat. The habitat can be saved if one of the communities acts unilaterally or by both communities sharing the costs of conservation. Because the habitat has the characteristics of a public good, if only one of the communities supports the habitat, both gain. Whether the habitat will be conserved depends on the costs relative to the benefits. If the costs of conserving the habitat are so high that the expense of a unilateral commitment outweighs the benefits the problem is a prisoners’ dilemma.

This scenario is represented in Matrix 3.8 where the pay-offs for two communities, Arleston and Waremouth, are the net benefits of conservation converted into monetised units. The value to each community of saving the habitat is 100 units. The cost of saving the habitat is 150 units. If the habitat is saved its benefits are non-excludable and non-rival and therefore both communities fully benefit regardless of who pays. If one community pays all of the 150 conservation costs, its net benefits are negative. If the costs are shared equally both communities gain. If neither community acts to save the habitat then neither gains and their net benefits are zero. What do you think will be the outcome of the conservation game represented in Matrix 3.8?
The conservation game is a prisoners’ dilemma for the two communities. Each community’s dominant strategy is not to conserve and therefore the dominant-strategy equilibrium is {not conserve, not conserve} even though both communities would be better off if they both conserved. This is the theoretical prediction of the outcome of the game. The conservation game illustrates how the free-rider effect impacts on the provision of public goods. It shows that if the parties who stand to gain from the provision of a public good act in their own self-interest the public good is unlikely to be supplied. Non-excludability and non-rivalry reduce the private incentives to contribute towards the provision of public goods and therefore intervention by government may be necessary to ensure their supply. The example of conservation was not chosen by accident. Many environmental problems such as pollution and threats to biodiversity are exacerbated because the benefits that derive from improvements in environmental quality are often both non-excludable and non-rival. Because of this private incentives to improve (or refrain from harming) the environment are weak. Environmental problems like pollution are the result.20

Ocean fisheries and the large tracts of tropical rain forest in South America and East Asia are effectively non-excludable resources since they are virtually impossible to police. Resources that are non-excludable in this way are called open-access resources. Open-access resources are not public goods since they are invariably rival. Fish caught by one group of fishermen cannot be caught again and once an area of forest has been logged it is unavailable to other would-be loggers (or any other users of the forest). When a resource is non-excludable but rival potential users face a prisoners’ dilemma but not in relation to supply, instead the issue is one of over-harvesting or over-exploitation. This problem was first analysed in relation to common land with open access grazing rights to local sheep farmers.21 For this reason the problem itself is often referred to as the ‘tragedy of the commons’.22 Fisheries, forests and grass for grazing are renewable resources but not all open-access resources are renewable. An example of a non-renewable open-access resource is a public
road. Roads are effectively non-excludable but they are definitely rival – traffic congestion provides ample evidence of that.23

The example considered here is that of ocean fisheries. Take a look at the fishing game represented in Matrix 3.9. In this game the players are two fishing fleets from two different countries, fleet Cody and fleet Kippen. The fleets are rivals for the stock of fish in the sea. Fishing yields per trawler are assumed to be higher per sailing the greater the stock of fish in the sea. Fishing costs will therefore be lower and profits higher, the greater the stock.24 The pay-offs in Matrix 3.9 reflect the profits from selling the fish that are caught over a fixed time period. Restrained fishing by both fleets generates a sustainable yield of fish and a reasonable level of profits for both fleets. Non-excludability implies that if fishing is unrestrained a fleet will trawl as long as there are positive profits to be made from fishing. Rivalry means that unrestrained fishing by one or both fleets depletes the stock of fish in the sea, lowers yields, raises fishing costs and lowers profits for both fleets. If one fleet shows restraint but the other does not the yields and profits of the fleet showing restraint will be lower than if neither or both had showed restraint. The yields and profits of the fleet not showing restraint will be higher.

Matrix 3.9  Fishing game

<table>
<thead>
<tr>
<th>Fleet Cody</th>
<th>Fleet Kippen</th>
</tr>
</thead>
<tbody>
<tr>
<td>restrained fishing</td>
<td>100, 100</td>
</tr>
<tr>
<td>unrestrained fishing</td>
<td>150, 25</td>
</tr>
</tbody>
</table>

If Cody and Kippen want to maximise their profits their dominant strategy is to fish indiscriminately. In the long term this may lead to non-sustainable yields and over-harvesting of the fisheries, possibly to extinction if the stock of fish is harvested beyond its critical minimum size (the level at which reproduction rates are so low that the stock is non-viable). Yet both fleets could make higher profits (probably for longer) if they could somehow agree to show restraint. The problem is a prisoners’ dilemma for the fleets. By acting in their own self-interest they both are worse off than if they had managed to cooperate.

The prisoners’ dilemma in ocean fisheries arises because access to the resource is open or non-excludable. The dilemma could therefore be solved in principle by restricting access. This may be easier in some situations than others. For example, property rights to fisheries closer to shores and where only a limited number of countries are affected should be easier to establish. One example where fishing rights have been restricted by quotas and more sustainable fishing practices have been the result is in Port Lincoln in South Australia. In this remote corner of South Australia there are no international border disputes to worry about and a combination of restricted access and self-regulation has
generated high incomes for licensed fisherman and sustainable stocks of bluefin tuna, rock lobster and king prawn.25

3.7 **Macroeconomics**

Prisoners' dilemmas can also arise in the macroeconomic environment when the actions of individual agents acting in their own self-interest have damaging effects on the macroeconomy. When this is a possibility acting in what appears to be self-interest can be self-defeating. Consider the case of a trade union leader negotiating a wage increase. From the perspective of the union leader it makes sense to try to secure a wage increase for the union membership that is as large as possible. The problem for the trade union leader is that if other trade union leaders act in the same way, implying a wages free-for-all, the overall negative effects on the economy in terms of rising inflation or higher unemployment are likely to outweigh the positive effects of any given wage increase. This situation is illustrated in the wages game shown in Matrix 3.10. The pay-offs in the wages game are the utility pay-offs of the trade union leaders. Their utility depends on the welfare of their members and this depends on the real value of their wages and whether they have a job or not.26

In the wages game one of the players (TU leader 1) is a representative leader of a major trade union in a national labour market. The trade union leader chooses between making either high or moderate wage demands. Although each trade union leader in the economy acts independently their decisions impact on each other. Because all the leaders of the major trade unions are in an identical position each of them is effectively playing against a collective of all the others. This is modelled by letting the ‘other’ player in the game be a conglomeration of all the other trade union leaders. This is a useful simplification when there are more than two players in a game but, in terms of their strategies and relative pay-offs, they are identical.

**Matrix 3.10  Wages game**

<table>
<thead>
<tr>
<th>TU leader I</th>
<th>all other TU leaders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>high wage demands</td>
</tr>
<tr>
<td>high wage demands</td>
<td>-5, -5</td>
</tr>
<tr>
<td>moderate wage demands</td>
<td>-10, 15</td>
</tr>
</tbody>
</table>
In this instance the game is an n-player prisoners’ dilemma. If all the leaders make high wage demands this triggers an upward inflationary spiral in the economy or massive redundancies or both. Either is disastrous for workers. If this is the only alternative the workers are better off if all the unions show restraint. But if one trade union leader makes a high wage demand while all the others show restraint the members of the first trade union benefit from high wage increases. The inflation this triggers leaves all the other union members worse off. Similarly, if one trade union leader shows restraint while all the others make high demands the economy still suffers but the employed members of the first trade union are not compensated by higher wages, so they are worse off. Unfortunately for the economy as a whole it is rational for every leader to go for high wages. This is not in their collective interests but to secure moderate wage demands all round requires some kind of deal on mutual restraint. The question then is how, if at all, could such a deal be instigated?

The wages game shows that where there are many players in a game, the interaction between them can still constitute a prisoners’ dilemma. In a prisoners’ dilemma, actions motivated by self-interest are not mutually beneficial, they are mutually harmful. Consequently, when interactions are characterised by a prisoners’ dilemma Adam Smith’s invisible hand may require some assistance in order to achieve a socially desirable outcome.

### 3.8 Resolving the prisoners’ dilemma

One of the questions addressed in the vast literature on the prisoners’ dilemma relates to evidence of collusion and cooperative behaviour in situations that can be characterised as prisoners’ dilemmas. Clearly such behaviour contradicts the theoretical prediction. For example, large firms can and do collude. If they did not, there would be no rationale for governments and supranational organisations like the EU to regulate these kinds of activities by firms. Clearly there is a perceived need for this type of regulation as embodied by antitrust policy in the USA, as enforced by the Office of Fair Trading and the Competition Commission in the UK and as encompassed in Article 81 of the European Community Treaty of Amsterdam.

In addition, there is considerable experimental evidence to suggest that people playing one-shot prisoners’ dilemma games will cooperate at least some of the time. In the experiments that have been conducted, of which there have been a large number, subjects playing one-shot prisoners’ dilemma games have been found to cooperate about half of the time (Camerer, 2003: 46). Similarly, subjects playing one-shot public good games have been shown to exhibit a systematic tendency not to free ride (Ledyard, 1995: 121). Changes in the relative pay-offs so that the pay-off from unilateral defection is less or the pay-off from unilateral (and multilateral) cooperation is more both increase the chances of cooperation in prisoners’ dilemma games and, equivalently, the rate
of provision in public good games. Communication between the players can also raise the rate of cooperation or contribution. Evidence of this kind contradicts the theoretical predictions in the same way as cooperative behaviour observed in the real world, outside the laboratory.

How then can such behaviour be explained other than by dismissing it as irrational? A number of possible answers to this question have been suggested in the academic literature. First of all it may be possible for the players to make enforceable or binding agreements to secure the cooperative outcome. Agreements could be enforced by the threat of punishment, possibly through a third party as discussed at the end of Section 3.1. Punishments could also be imposed through the legal system if for instance contracts are broken, or through government imposed penalties. Threats that work through informal networks of associates can also be effective. When threats to punish are credible they lower the pay-offs from non-cooperative behaviour. If the punishments are hard enough (so that in Matrix 3.3 c < a and d < b) then they can make cooperation a dominant strategy. In this case there is no dilemma.

Second, if a prisoners’ dilemma is repeated then, intuitively, the players should have more incentive to cooperate as their pay-offs are collected not just once but however many times the game is played. Repetition means that players need to choose long-term strategies that take into account their future pay-offs. They also have time to learn about the game and each other. If there are enough repetitions of the game then the possibility of higher pay-offs in the future as a result of earlier cooperative behaviour could outweigh the short-term gains from defection. This is what some analysts refer to as ‘the shadow of the future’ influencing decisions made today.

Lastly, if one or both of the players is unsure about the other’s pay-offs then this could also change the outcome of the game. If, for instance, one of the players is not sure that non-cooperation is a dominant strategy for the other then it could make sense for the first player to choose the cooperative strategy but only if the prisoners’ dilemma is repeated.

All of these possibilities point to ways of resolving prisoners’ dilemmas without weakening the strong rationality assumptions that are integral to game theory. They can also help to reconcile the game theoretic predictions with both real-life observations and experimental evidence. All of them are given attention later in this book. The idea of a credible threat is developed in Chapter 4 and the possibility of making binding contracts is discussed in Chapter 9. Repeated prisoners’ dilemma games are analysed in Chapter 8.

**Summary**

This chapter has focused on just one game, the prisoners’ dilemma. The dominant-strategy equilibrium of the prisoners’ dilemma is not Pareto-efficient as both
players could do better by choosing their dominated strategies. This equilibrium is unsettling because it suggests that rational play can be self-defeating which raises some interesting questions about the definition of rationality used in game theory. After all, how rational is rational if non-rational choices result in higher pay-offs? The assumption that human beings are motivated only by self-interest can also be criticised. If this were a true reflection of human nature it would be difficult to explain why people give to charity, leave tips in restaurants they are unlikely to revisit, look after their children or care for their elderly relatives. This kind of behaviour suggests that people are not selfish all the time – but of course they are not altruistic all the time either.

Rabin (1993) suggests instead that people engage in a type of reciprocal fairness: they are nice to people who are nice to them, but not so nice to people who are unkind to them. This idea can be incorporated into game theory by adding fairness bonuses or subtracting penalties from pay-offs. In the prisoners’ dilemma fairness bonuses raise both players’ pay-offs when they both cooperate and unfairness penalties lower the pay-offs of a player who defects when the other cooperates. If the fairness bonus is high enough the dominant strategy equilibrium of a prisoners’ dilemma adjusted in this way is for both players to cooperate (see Camerer and Thaler, 2003: 162). Thus allowing for a shared sense of fairness can resolve the one-shot version of the prisoners’ dilemma. But people do not always want to be nice to each other or expect other people to be nice to them and in these cases the prisoners’ dilemma is less likely to be resolved by shared beliefs about fairness. Nevertheless, behavioural approaches of this kind offer some interesting answers to the questions raised by the prisoners’ dilemma and game theory more generally. These questions are worth addressing because, as you have seen, there are numerous applications of the prisoners’ dilemma – it is not restricted to prisoners.

3.1
One possibility is the one illustrated in Matrix 3.11:

Matrix 3.11

<table>
<thead>
<tr>
<th></th>
<th>cooperate</th>
<th>defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>cooperate</td>
<td>6, 6</td>
<td>1, 8</td>
</tr>
<tr>
<td>defect</td>
<td>8, 1</td>
<td>5, 5</td>
</tr>
</tbody>
</table>
The game represented in Matrix 3.11 is a prisoners’ dilemma because the dominant strategy of both players is to defect yet in the dominant-strategy equilibrium \{\text{defect, defect}\} the players’ pay-offs are less than if they had both cooperated.

### 3.2

**Matrix 3.7 International trade 3**

<table>
<thead>
<tr>
<th></th>
<th>Little Rosatia</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no tariff</td>
<td>impose tariff</td>
</tr>
<tr>
<td>Greater Jesmania</td>
<td></td>
<td></td>
</tr>
<tr>
<td>no tariff</td>
<td>10, 10</td>
<td>-1, 8</td>
</tr>
<tr>
<td>impose tariff</td>
<td>15, -1</td>
<td>2, -2</td>
</tr>
</tbody>
</table>

In international trade 3 the dominant-strategy equilibrium is for Little Rosatia not to impose a tariff and for Greater Jesmania to impose a tariff. This suggests that in trading relations between small and large countries tariffs are more likely to be raised by the latter. An example is the tariff wall erected by the EU against agricultural imports from smaller, developing countries as part of its Common Agricultural Policy (CAP).

### Problems

1. In the pay-off matrix below use numbers between \(-8\) and \(-2\) to write pay-offs for Alf and Bert such that \{\text{confess, confess}\} is a dominant-strategy equilibrium but not a Pareto-efficient one. Is the game you have created a prisoners’ dilemma? If so explain why and if not explain why not.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Bert</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>hold out</td>
<td></td>
</tr>
<tr>
<td>Alf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>hold out</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Imagine that the prisoners playing the prisoner’s dilemma game represented in Matrix 3.1.1 (p. 59) have secured the services of a professional hit man in an attempt to enforce an agreement to deny. Show in a pay-off matrix how this could affect the prisoners’ pay-offs? Is the game you have constructed still a prisoners’ dilemma?
Questions for discussion

1 Describe three or more examples of prisoners’ dilemmas that are faced by real people (acting individually or in groups) in real life.

2 How, if at all, are the prisoners’ dilemma problems, described in the examples you have outlined in answer to Problem 1, resolved? If they are not resolved in practice how do you think they might be resolved?

3 In what sense is the Nash equilibrium of the prisoners’ dilemma unsatisfactory?

Answers to problems

1 The pay-off matrix below shows one possibility. The dominant-strategy equilibrium is \{confess, confess\} but it is Pareto-dominated by \{hold out, hold out\}. The game in the matrix is a prisoners’ dilemma as the pay-offs satisfy the conditions that \(c > a > d > b\) where \(c = -2\), \(a = -3\), \(d = -6\) and \(b = -8\).

<table>
<thead>
<tr>
<th></th>
<th>Alf</th>
<th>Bert</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>hold out</td>
<td>confess</td>
</tr>
<tr>
<td>hold out</td>
<td>-3, -3</td>
<td>-8, -2</td>
</tr>
<tr>
<td>confess</td>
<td>-2, -8</td>
<td>-6, -6</td>
</tr>
</tbody>
</table>

2 Matrix 3.1.1 is shown again below. If the prisoners secured the services of a hit man who agreed, for a fee, to kill a close relative of any one of them who confessed (whether the other confessed or not) the pay-offs could look like those in Matrix 3.1.2. Note that the pay-offs in Matrix 3.1.2 take into account more than the length of the possible prison sentences. They also incorporate how the prisoners’ might feel about the death of their relative. I am assuming that this makes them very unhappy. The pay-offs in Matrix 3.1.2 make \{deny, deny\} the dominant-strategy equilibrium of the game. This is a Pareto-efficient outcome and the game is no longer a prisoners’ dilemma even though it is still being played by prisoners.
Prisoners' dilemma

**Matrix 3.1.1** Prisoners' dilemma

<table>
<thead>
<tr>
<th></th>
<th>prisoner 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>deny</td>
<td>-1, -1</td>
</tr>
<tr>
<td>confess</td>
<td>0, -10</td>
</tr>
</tbody>
</table>

**Matrix 3.1.2** Hiring a hit man to resolve the prisoners' dilemma

<table>
<thead>
<tr>
<th></th>
<th>prisoner 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>deny</td>
<td>-1, -1</td>
</tr>
<tr>
<td>confess</td>
<td>-100, -10</td>
</tr>
</tbody>
</table>

**Notes**

1. Or the prisoner's dilemma, which one seems to be a matter of personal preference but as there are two prisoners and the dilemma is shared by both, the former representation is used here.

2. For a detailed discussion see, for example, Hargreaves Heap (1989) or, specifically in relation to the prisoners' dilemma, Rapoport (1974).

3. The prisoners' dilemma game is attributed either to Tucker (1950) or Flood (1952). See Roth (1995a: 87 note 12) for a brief discussion of the origins of the prisoners' dilemma and a review of initial experiments with prisoners' dilemma games. See Mirowski (2002: 357–60) for a full discussion.

4. Or collectively rational behaviour which as defined by Rapoport (1974: 18) is behaviour that prescribes a course of action to both players simultaneously. In a prisoners' dilemma collectively rational behaviour would result in both players being better off than if they had acted in their own self-interest that is in accordance with individual rationality.

5. Specifically non-cooperative game theory. The distinction between cooperative and non-cooperative game theory is returned to in Chapter 9.

6. Simply assuming that the players can make agreements that are truly binding changes the game from a non-cooperative one to a cooperative game. Cooperative games of this kind are discussed in Chapter 9. You should try not to confuse the idea of a cooperative game as defined in Chapter 1, Section 1.6 with the idea of cooperation as a strategy option for the players in a prisoner's dilemma as shown in Matrix 3.3.

7. Usually the pay-offs in a prisoners' dilemma are symmetric (the game is the same from the perspective of either player) but a game can still be a prisoners' dilemma even if it is not symmetric. All that is required is that each player's pay-offs satisfy the inequalities $c > a > d > b$ in Matrix 3.2.

8. Or countries as in the case of the oil cartel formed by the Organisation of Petroleum Exporting Countries (OPEC).
When firms collude to maximise their joint profits they are effectively acting as a monopoly and are therefore able to extract higher (monopoly) profits from the industry.


The European Commission made its ruling on 30 October 2002 see http://europa.eu.int/comm/archives.

If the collusive agreement is one that maintains a high price by restricting output and a large firm raises output by a significant proportion, the consequent fall in the market price could hurt the defector as much as its competitor. In this situation the large firm has no incentive to break the collusive agreement and even though a small firm has, the larger firm may be able and willing to compensate the other in order to sustain the agreement.

Or supranational confederations like the European Union.


Trade implies the exchange of one product for another thus the tariffs imposed by Jesmania and Rosatia would be on different commodities. In reality, unless either Rosatia or Jesmania has a monopoly in one or other of the traded commodities, a tariff imposed by either of them would affect exporters in other countries. For simplicity the pay-offs and strategies of these countries are ignored.

Beneficial terms of trade effects can arise when there is a reduction in the price of the imported good as a result of reduced demand due to the tariff. This positive effect is likely to be more significant for larger countries because a fall in import demand in a country like the USA, for instance, is likely to have a greater (downward) influence on world prices than an equivalent fall in a country like Lithuania (see the literature on optimal tariffs, e.g. Venables 2003: 412–13).

See a microeconomics text book such as Pindyck and Rubinfeld (2001: Chapter 18) for a fuller discussion of public goods.

Wide-ranging benefits that extend beyond the individual consumer of a good or service (such as education or health) are known as positive externalities. In the case of education and health the benefits of an educated and healthy workforce extend beyond the individual worker to the rest of society. These kinds of benefits are both non-rival and non-excludable.

Public goods may also be analysed as a chicken game (Ledyard, 1995: 144–5) or a stag hunt game (Camerer, 2003: 377).

See an environmental economics text such as Field and Field (2002: Chapter 4) for a more detailed discussion of these issues.

Resources like these, with group access rights are often referred to as common property resources.

The term was popularised by Hardin (1968).

See an environmental economics text such as Hanley, Shogren and White (2001: Chapter 7) or Van Kooten and Bulte (2000) for a more detailed analysis of open access resources.

If there are more fish in the sea they are easier, quicker and therefore cheaper to catch. As long as prices do not fall in line with costs as catches increase, profits per catch will be higher.


See a labour economics text such as Sapsford and Tzannatos (1993: Chapter 10) or a text on the economics of trade unions such as Booth (1996) for a more detailed discussion of trade union utility functions.

See www.oft.gov.uk or www.competition-commission.org.uk.

Originally Article 85 of the Treaty of Rome. See, for example, Martin (2001).
29 Many of the prisoners’ dilemma games that subjects are asked to play in experiments are repeated games (see Roth, 1995a: 27). This evidence is discussed in Chapter 8.

30 Mirowski (2003: 458) quotes Simon (1982: 2,487–8) who states that ‘the main product of the very elegant apparatus of game theory has been to demonstrate quite clearly that it is virtually impossible to define an unambiguous criterion of rationality for this class of situations’.

31 For an introduction to this debate see, for example, Frank (2003: Chapters 7 and 8). For a discussion on economic rationality in relation to a specific example see Basu (2003: 896–7).
TAKING TURNS

Concepts and techniques

- Sequential moves
- Dynamic games
- Subgame perfect Nash equilibrium
- Backward induction
- Credible threats
- Extensive forms, game trees.

After working through this chapter you will be able to:

- Analyse games in which the players move sequentially
- Explain the difference between simultaneous and sequential moves and use extensive forms or game trees to illustrate sequential games
- Explain why moves might not be the same as strategies in dynamic games
- Complete strategic forms for sequential-move games
- Explain what is meant by a credible threat
- Show that sequential games can have Nash equilibria that are not supported by credible threats
- Explain what is implied by backward induction
In the games analysed in this chapter one player moves first and the other sees the first player’s move before making his or her move. Games where players move sequentially in this way are called sequential-move or dynamic games. In these kinds of games the concept of a Nash equilibrium as defined in Chapter 2 is not sufficient to ensure that players’ strategies prescribe moves that are best responses to each other at every decision point in the game. Remember that a player’s strategy for a game needs to map out their plan of action, their moves, for the entire game, taking into account all eventualities. Not all the eventualities will actually be realised. Which are, and which are not will depend on the moves of the players in the game. This means that a player’s strategy for the game may need to specify moves that are never actually made. Consequently a player can threaten (or promise) to make a move in order to secure a preferred outcome but if the other player takes the threat seriously they will not need to carry the threat out. However, a threat or a promise will only be credible if it would actually be carried out by a rational player if required to do so. A threat will be credible if it would be in a player’s best interest to carry it out in these circumstances. If a threat or a promise is not credible in this sense then it cannot be a best response to the other player’s move at that particular point in the game. An equilibrium strategy for the whole game needs to specify moves that are best responses at all stages of the game. Therefore, if a threat or a promise involves a move that is not a best response at some decision point in the game it cannot be part of an equilibrium strategy for the whole game.

In dynamic games, Nash equilibria as defined in the previous chapter that incorporate non-credible threats or promises can exist. Therefore the concept of a Nash equilibrium needs to be refined. The analysis in this chapter shows that the idea of a subgame perfect Nash equilibrium is a more appropriate equilibrium concept for games in which the order of moves matters since this refinement of Nash equilibrium rules out strategy combinations that involve non-credible threats. The method of backward induction is used to show this and to determine the subgame perfect Nash equilibrium of the games analysed.

In Sections 4.1 to 4.3 three different games with sequential moves are examined in detail. The concept of a subgame perfect Nash equilibrium is defined and backward induction is used to determine the subgame perfect Nash equilibrium
of each game. In Section 4.4 an entry deterrence game is analysed to explore some ideas relating to credibility and in Section 4.5 the centipede game is used to illustrate some of the limitations of the backward induction method.

4.1 Foreign direct investment game

The example developed in this section is called foreign direct investment (FDI). It is a variation on the foreign investment game you saw in Chapter 2. In this version of the game the two companies are Alpha and Beta. Alpha moves first and only Alpha is in a position to consider the option of making a foreign direct investment by opening a subsidiary in another country, Jesmania. Alpha is currently exporting to Jesmania and can continue to export for the next 10 years (the expected life of its product) or engage in FDI by opening a subsidiary. Exporting is less costly but leaves Alpha’s market share vulnerable to competition. If Alpha opens a subsidiary, by employing Jesmanians and developing links with the Jesmanian community it creates customer loyalty for its product and its market share is more secure. If there is no strategic threat from Beta then Alpha will choose the less costly option of exporting and will not engage in FDI. Alpha’s profits when there is no strategic threat are shown in Matrix 4.1 in billions of euros.

Matrix 4.1 Alpha’s profits with no strategic threat from Beta

<table>
<thead>
<tr>
<th>Alpha’s moves</th>
<th>FDI</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>export only</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>

Beta is not currently exporting but is considering expanding the market for its product by developing an export market in either Jesmania or at home. If it decides to export to Jesmania it will be in direct competition with Alpha and its profits will depend on whether Alpha is exporting or has chosen FDI. If Beta decides to export to Jesmania then Beta’s profits will be higher if Alpha has not directly invested in Jesmania. In this case Beta’s profits will also be higher than if it doesn’t export and simply expands its domestic market. But if Alpha invests directly in Jesmania then Beta cannot compete with Alpha. In these circumstances Beta will incur major costs if it tries to enter the market but will only secure a small market share as a result, making a net loss overall. Therefore when Alpha invests directly in Jesmania, Beta’s profits are higher if it doesn’t export and instead expands production at home. Beta’s pay-offs are shown in billions of euros in Matrix 4.2.
Matrix 4.2  Beta’s pay-offs (contingent on Alpha’s move)

<table>
<thead>
<tr>
<th>Beta’s moves</th>
<th>Alpha’s moves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FDI</td>
</tr>
<tr>
<td>export</td>
<td>-5</td>
</tr>
<tr>
<td>not export</td>
<td>10</td>
</tr>
</tbody>
</table>

If Beta decides to export to Jesmania then Alpha’s profits will also be lower. If Alpha is only exporting to Jesmania it has no alternative but to passively share its export market. If it has chosen direct investment then it engages in a costly campaign to retain its monopoly position. This campaign is partially successful in that Alpha remains the market leader in Jesmania but the challenge by Beta weakens its monopoly of the market by opening up the market to domestic and other foreign competition. Alpha’s pay-offs if Beta exports to Jesmania are shown in billions of euros in Matrix 4.3. With these pay-offs Alpha still prefers the export only option even if Beta enters the Jesmanian market.

Matrix 4.3  Pay-offs to Alpha if Beta exports to Jesmania

<table>
<thead>
<tr>
<th>Alpha’s moves</th>
<th>FDI</th>
<th>export only</th>
</tr>
</thead>
<tbody>
<tr>
<td>export only</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

If the firms moved simultaneously then the pay-off matrix for the game would look like the one in Matrix 4.4. In Matrix 4.4 {export only, export} is the only Nash equilibrium in pure strategies. Export only is a best response for Alpha to Beta’s move of exporting to Jesmania and if Alpha only exports to Jesmania then exporting to Jesmania is a best response for Beta. This Nash equilibrium seems to confirm that Alpha will choose the export only strategy regardless of whether there is a competitive threat from Beta or not. However, this way of representing the game ignores the sequence of moves.

Matrix 4.4  Pay-off matrix for FDI with simultaneous moves

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Beta</th>
<th>export</th>
<th>not export</th>
</tr>
</thead>
<tbody>
<tr>
<td>FDI</td>
<td></td>
<td>25, -5</td>
<td>40, 10</td>
</tr>
<tr>
<td>export only</td>
<td></td>
<td>30, 30</td>
<td>60, 10</td>
</tr>
</tbody>
</table>
In the FDI game Alpha actually moves first and chooses between FDI and export only. Beta moves last and chooses between exporting to Jesmania or not. But Beta sees Alpha’s move and therefore Beta’s choice of move is contingent on Alpha’s move. This means that there is an important difference between the strategies of Alpha and Beta. Remember that a strategy is a plan for playing a game; it should give a complete description of what a player plans to do during the game and the more complex the game the more detailed a player’s plan needs to be.

Because Alpha moves first, its plan for playing the game only needs to specify its preferred choice between FDI and export only. Alpha’s available strategies are therefore the same as its available moves: a straight choice between FDI or export only. For Beta the situation is more complex. Beta’s plan for the whole game needs to specify moves that are contingent on Alpha’s choice. This is easiest to see in the extensive form or game tree for the FDI game shown in Figure 4.1.

In the game tree Alpha moves first at the decision node labelled A and chooses between FDI and export only. If Alpha chooses FDI the game moves to B1 where Beta chooses between export and not export. If Alpha chooses not export at the decision node labelled A the game moves to B2 where again Beta decides between export and not export. The extensive form shows the pay-offs of the firms written at the terminal nodes. It should be clear from the way the game tree is drawn that while Alpha simply chooses between FDI and export only Beta’s choices are more complicated. A strategy for Beta needs to specify a choice at both B1 and B2 as Beta can’t be sure before Alpha moves what Alpha will choose; Beta needs to have a full set of contingency plans.

More specifically, a strategy for Beta needs to specify what Beta’s move should be if Alpha chooses FDI and what it should be if Alpha chooses export only. Because a strategy for Beta needs to map out a plan for all eventualities Beta actually has four possible strategies from which to choose:

![Figure 4.1 Extensive form or game tree for FDI](image-url)
1. Export to Jesmania whatever Alpha does (export, export).

2. Not export to Jesmania whatever Alpha does (not export, not export).

3. Export if Alpha chooses FDI and not export if Alpha chooses export only (export, not export).

4. Not export if Alpha chooses FDI and export if Alpha chooses export only (not export, export).

If you are finding it difficult to understand why Beta has four strategies instead of two, imagine that the boss of Beta has to go into hospital for an operation and leaves her deputy in charge of the firm. Alpha is expected to make its decision about whether to undertake FDI while Beta’s boss is in hospital. Beta’s boss wants to leave her deputy with precise instructions about what to do in response to Alpha’s choice when Alpha makes it. She writes down her chosen strategy to cover all contingencies and expects the deputy to carry out her instructions precisely. If she writes down simply export or not export, the deputy will either export or not export whatever Alpha does. Simply writing export or not export corresponds to export whatever Alpha does and not export whatever Alpha does, that is (export, export) and (not export, not export). If Beta’s boss wants to do anything different then she will have to be more specific. For example, if she wants Beta to export if Alpha chooses FDI but not export if Alpha chooses export only then she can write this down as (export, not export). Alternatively, if she wants Beta to not export if Alpha chooses FDI but export if Alpha chooses export only, then she can write this as (not export, export) as shown. Thus Beta’s boss needs to write down one of four possible strategies.

In Matrix 4.4 the pay-offs correspond only to the firms’ moves and for Beta these are not the same as its fully defined strategies. In dynamic games where one player’s moves are conditional on another’s this will always be the case and therefore a pay-off matrix corresponding to moves – a move matrix – is not really the strategic form of the game as it does not accurately depict the strategy choices of the players. It follows that any Nash equilibrium found by indicating best responses to moves alone is likely to be misspecified. Matrix 4.5 shows the fully specified pay-off matrix for FDI where the players’ choices are between strategies rather than moves (although these are the same for Alpha). In Matrix 4.5 Beta has four strategies from which to choose while in the move matrix it has only two: export to Jesmania corresponding to (export, export) and not export to Jesmania corresponding to (not export, not export). This is because Matrix 4.4 does not take account of the sequence of moves and therefore only specifies strategies for Beta that involve making the same move whatever Alpha does.
The two Nash equilibria in Matrix 4.5 are \{export only, (export, export)\} and \{FDI, (not export, export)\}. In the first of these, Beta chooses export whatever Alpha chooses and Alpha chooses export only. In the second equilibrium Beta chooses not export if Alpha chooses FDI and export if Alpha chooses export only. As Alpha chooses FDI, Beta chooses not export. The first Nash equilibrium corresponds to the Nash equilibrium found in the simple moves matrix: \{export only, export\}. The second Nash equilibrium is not represented in the moves matrix. Can you see that the second Nash equilibrium, \{FDI, (not export, export)\}, is preferred by Alpha while the first is preferred by Beta? In the first Nash equilibrium Alpha’s pay-off is 40, in the first it is only 30. In the first Nash equilibrium Beta’s pay-off is 30, in the second it is only 10.

You should also be able to see that in the strategic form different strategies by Beta can lead to the same combination of moves and pay-offs for both players depending on what Alpha does. For example, \{FDI, (export, export)\} and \{FDI (export, not export)\} both result in Alpha undertaking FDI and Beta exporting. This gives Alpha a pay-off of 25 and Beta a pay-off of \(-5\). Beta’s strategies are different but the outcome is the same. This is one reason why it is important to think about an equilibrium in terms of the players’ strategies rather than their pay-offs since the same pay-offs can result from different strategy pairs.

Now take a look at the two Nash equilibria that have been identified in the strategic form of the game. Do you think that both are equally feasible? Or do you think that either or both of them could embody moves that are not credible because they are not best, that is Nash responses to a move by the other player at some point in the game? In game theory this question is answered by identifying whether the Nash equilibria are also subgame perfect Nash equilibria. By definition, the players’ strategies in a subgame perfect Nash equilibrium specify moves that are best responses at all the decision points or nodes in the game.
To find a subgame perfect Nash equilibrium for this game we can use backward induction.\(^2\) Backward induction is used to choose between multiple Nash equilibria by checking that the players’ moves are best responses to each other at every decision node. This process often amounts to checking the credibility of threats. To use backward induction start at an end point or terminal node of the game in its extensive form and work back through the game analysing sub-sets or subgames of the whole game independently. A subgame is a subset of the whole game that starts at some decision node where there is no uncertainty\(^3\) and branches out from that node. Subgames end at nodes that are terminal nodes of the whole game.\(^4\) In the FDI game there are two proper sub-games: the subgame beginning at B\(_1\) in Figure 4.1 and the subgame beginning at B\(_2\). After identifying the subgames you can check if the players’ strategies specify moves that are best responses in every subgame. If they are not, then a player’s threat or promise to make such a move is not credible so can be ignored. Only threats or promises that are in a player’s self-interest are credible. (If you are still unsure about the idea of a subgame you can test your understanding in Problem 1 at the end of this chapter.)

**A subgame**

- A piece of a game that begins at a decision point where there is no uncertainty and ends at decision nodes that are terminal nodes of the whole game.

For a Nash equilibrium to be subgame perfect it has to specify a combination of credible moves in every subgame: moves that are best responses at every decision node. In the actual equilibrium that is played out some decision nodes will not be reached. Such nodes are said to be off the equilibrium path of the game. But players still need to specify their moves at these points as threatened actions off the equilibrium path influence other players’ strategy choices on it. In a subgame perfect Nash equilibrium any threat to follow a given strategy, in order to enforce a particular strategy choice by other players, needs to be credible. To be credible a threat must be in a player’s best interest to carry out if called upon to do so.\(^5\) Backward induction involves working back through the game checking that the players’ strategies specify moves that constitute a Nash equilibrium in every subgame. If the players’ strategies are best responses in every subgame then they are playing rationally by acting in their own self-interest throughout the game.
4.1.1 Using backward induction to find the subgame perfect Nash equilibrium of the FDI game

In the FDI game there are two proper subgames: the subgame beginning at $B_1$ in Figure 4.1 and the subgame beginning at $B_2$. Using backward induction in this case means working back from the terminal nodes to the subgames beginning at $B_1$ and $B_2$ and checking that Beta's strategy specifies moves that are best responses to Alpha's move. We can check each subgame in turn.

**The subgame beginning at $B_1$**

In order to have reached the subgame beginning at $B_1$ Alpha must have chosen FDI. At $B_1$ Beta chooses between export and not export. If Beta chooses not export Beta's pay-off is 10. If Beta chooses export Beta's pay-off is $-5$.

Consequently, not exporting is Beta's best response at $B_1$ to FDI by Alpha and exporting is not a best response. It is not rational for Beta to choose export if Alpha chooses FDI because by exporting Beta's pay-off is less than it would be if it chose not to export to Jesmania. Therefore if Alpha chooses FDI and Beta is rational, Beta will choose not export, as not exporting is a best response to FDI by Alpha. This means that any threat by Beta to export if Alpha chooses FDI is not credible. Because Beta's pay-offs are common knowledge Alpha knows this. It follows that Alpha knows that if it chooses FDI its pay-off will be 40. We can illustrate this in the game tree by highlighting the relevant branches of the tree as I have done in Figure 4.1.1. In Figure 4.1.1 the thickened branches show that if Alpha chooses FDI Beta's best response at $B_1$ is not to export.
In order for the game to have reached B2, Alpha must have chosen export only. At B2 Beta chooses between export and not export. If Beta chooses export Beta’s pay-off is 30. If Beta chooses not export Beta’s pay-off is 10.

At B2 choosing export is a rational response to Alpha’s move. By exporting Beta’s pay-off is 30 and otherwise it is only 10. Therefore Beta will choose export at B2 and Alpha’s pay-off will be 30. Alpha knows this and so can predict that if it chooses export only its pay-off will be 30. This is indicated in Figure 4.1.2 where the thickened branches show that if Alpha chooses not export Beta’s best response at B2 is to export.

The backward induction procedure shows that Beta’s best response at B1 is not export and at B2 it is export. This implies that (not export, export) is Beta’s only rational strategy. Only this strategy can be part of a subgame perfect Nash equilibrium and we can rule out all Beta’s other alternatives. This is shown in Figure 4.1.3 where the thickened branches show Beta’s best responses at each of Beta’s decision nodes.

![Figure 4.1.2 Beta's best response at B2](image)

**The subgame beginning at B2**

![Figure 4.1.3 Beta's best responses at B1 and B2](image)
**Alpha’s choice at A**

Having analysed Beta’s choices it is now possible to work back from B₁ or B₂ to the beginning of the game where Alpha is choosing between FDI and export only. Common knowledge means that Alpha knows that if it chooses FDI Beta will choose not export and Alpha’s pay-off will be 40 (following the thickened branch from B₁ in Figure 4.1.3). But if Alpha chooses export only Beta will choose to export as well and Alpha’s pay-off will only be 30 (following the thickened branch from B₂ in Figure 4.1.3). Thus FDI is a best response by Alpha to Beta’s only credible strategy of (not export, export). This implies that the only subgame perfect Nash equilibrium of the FDI game is \{FDI, (not export, export)\} as illustrated by the thickened branches in Figure 4.1.4.

\{FDI, (not export, export)\} is the only Nash equilibrium in which Beta’s strategy specifies moves that are best responses in both subgames. It is therefore the only subgame perfect Nash equilibrium of the game. This is Alpha’s preferred Nash equilibrium outcome and Beta’s least preferred. It appears that in this game Alpha has a first-mover advantage since Alpha’s costly FDI strategy deters Beta from entering the Jesmanian market. Alpha’s commitment to FDI is rational as Beta’s implicit threat to export if Alpha commits to FDI is not credible.

In the next section a more abstract dynamic game is examined in order to highlight a number of different possibilities in games like FDI. In Sections 4.3 and 4.4 further examples are examined. The structure of these games is different but the method for finding the subgame perfect Nash equilibrium by ruling out non-credible threats or promises is the same.

---

**4.2 Nice—not so nice game**

In this section we examine another sequential move game. It is similar to the FDI game in that there are two players and one moves first. However, the players’
moves are labelled according to whether they are potentially more or less advantageous for the other player. Although the impact of one player’s choice on the other’s pay-off is made explicit, the players are still rational and self-interested and therefore choose their strategies in their own best interests. However, labelling the moves in this way highlights the threat–promise nature of the players’ strategies which is an inherent feature of many sequential move games.

In this game there are two players: Players One and Two. Player One moves first and he chooses between two moves. One of these is potentially more advantageous for Two because if One chooses this move Two will have the chance to ensure her highest possible pay-off (in the discussion that follows it helps if we assume One is male and Two is female). If One chooses this move we can say that he is being nice to Two and if he does not then he is being not so nice to Two. Thus he has a choice between two moves: nice (to Two) and not nice (to Two). Two moves second after seeing One’s move. Whether One has chosen his nice strategy or not Two makes a choice between two moves one of which is relatively more advantageous for One. Thus Two similarly chooses between a nice (to One) and a not so nice (to One) move. Although the impact of one player’s move on the other’s possible pay-off is common knowledge neither player cares about the other’s pay-off, only their own. To summarise, One moves first and chooses between nice and not so nice. Two sees One’s move and chooses between nice and not so nice.

Because One moves first, his strategies correspond to his moves as he has a simple choice between nice and not so nice. Because Two moves after One her strategies are more complex as they are contingent on One’s move. Two has four possible strategies:

1. (nice, nice): always choose nice.
2. (not so nice, not so nice): always choose not so nice.
3. (nice, not so nice): choose nice if One chooses nice; choose not so nice if One chooses not so nice.
4. (not so nice, nice): choose not so nice if One chooses nice; choose nice if one chooses not so nice.

The extensive form for this version of nice–not so nice is shown in Figure 4.2. The fully specified pay-off matrix is shown in Matrix 4.6.

The extensive form shows clearly that there is conflict in this game. Two would prefer One to choose nice at decision node 1 so that she can secure her maximum pay-off of 6. However, One achieves his maximum pay-off of 5 by choosing not so nice as long as Two chooses nice at 2B. But Two may be able to deter One from choosing not so nice by threatening to choose not so nice at 2B and by promising to choose nice if One also chooses nice. But is this threat and promise strategy by Two credible?
An examination of the strategic form in Matrix 4.6 shows that \{nice, (nice, not so nice)\} is the only Nash equilibrium of the game (you should check this by highlighting the best response pay-offs of both players). But is \{nice, (nice, not so nice)\} also a subgame perfect Nash equilibrium?

To answer this question we can use backward induction to work back from the terminal nodes in Figure 4.2 to the subgames beginning at 2A and 2B. By doing this we can check whether (nice, not so nice) is potentially a subgame perfect Nash equilibrium strategy for Two. At 2A Two’s pay-off is 6 if she chooses nice and 0 otherwise. Since 6 > 0 nice is her best response to One’s choice of nice at decision node 1. At 2B Two’s pay-off is 2 if she chooses nice and 3 otherwise. Since 3 > 2 her best response to One’s choice of not so nice is to similarly choose not so nice. This implies that (nice, not so nice) is entirely rational for Two and I have highlighted the corresponding branches of the game tree in Figure 4.2.1.

Since (nice, not so nice) is a rational strategy for Two her threat to play not so nice if One chooses not so nice is credible and her promise to play nice if One chooses nice can also be trusted. If we can work this out so can One. One will assume that if he chooses nice at decision node 1 his pay-off will be 2. If he chooses not so nice his pay-off will be –6. Therefore One will choose nice. One’s choice of nice is a best response to (nice, not so nice) by Two and therefore \{nice (nice, not so nice)\}, the Nash equilibrium identified in the strategic form, is the only subgame perfect Nash equilibrium of the game. The subgame perfect Nash equilibrium defines the players’ moves through the game along the equilibrium path as illustrated in Figure 4.2.2 by the thickened branches.

### Figure 4.2

**Extensive form for nice—not so nice 1**

<table>
<thead>
<tr>
<th></th>
<th>nice, nice</th>
<th>not so nice, not so nice</th>
<th>nice, not so nice</th>
<th>not so nice, nice</th>
</tr>
</thead>
<tbody>
<tr>
<td>nice</td>
<td>2, 6</td>
<td>-1, 0</td>
<td>2, 6</td>
<td>-1, 0</td>
</tr>
<tr>
<td>not so nice</td>
<td>5, 2</td>
<td>-6, 3</td>
<td>-6, 3</td>
<td>5, 2</td>
</tr>
</tbody>
</table>

**Matrix 4.6** Strategic form for nice—not so nice 1

An examination of the strategic form in Matrix 4.6 shows that \{nice, (nice, not so nice)\} is the only Nash equilibrium of the game (you should check this by highlighting the best response pay-offs of both players). But is \{nice, (nice, not so nice)\} also a subgame perfect Nash equilibrium?
Because Two’s threat to play not so nice at 2B is a credible threat, Two does not actually have to play not so nice in the equilibrium. One chooses nice and decision node 2B is never reached. But Two’s threat to play not so nice at 2B is still part of her equilibrium strategy and it induces One’s choice of nice. This shows how threatened moves off the equilibrium path can support a subgame perfect Nash equilibrium but to do so they need to be credible.

**Exercise 4.2**

The extensive form of a different version of the nice—not so nice game, nice—not so nice 2, is shown in Figure 4.2.3. What is the subgame perfect Nash equilibrium of nice—not so nice 2? In this version of the game is Two’s threat to play not so nice at 2B credible? If so, why, and if not, why not? Does One gain any advantage by moving first in this game? Did One gain any advantage by moving first in nice—not so nice 1?
The two players in this game are a landowner, Bert, and a hiker, Angela. Bert owns some land by a river in a beautiful part of the English countryside. Angela likes to ramble in the countryside and would like to walk through Bert’s land beside the river instead of walking along the tarmac road around Bert’s land. Bert doesn’t want walkers on his land and he moves first by putting up a large sign threatening to prosecute trespassers who come onto his land. Angela sees the sign and chooses between trespassing on Bert’s land or not. If she defers to Bert’s threat by not walking over his land he is satisfied but she is not. If she doesn’t cross his land Bert doesn’t prosecute, he effectively does nothing except perhaps smugly repaint his sign. If Angela challenges his threat to prosecute by crossing Bert’s land Bert then has to choose between carrying out his threat to prosecute or not. If he prosecutes the law is such that if Angela has merely walked over his land he loses – in England there is no criminal law against trespassing unless the trespasser commits criminal damage of some kind. Assuming Angela doesn’t commit any criminal damage, the whole procedure will be a waste of time and money for both of them. The players’ pay-offs for this game are shown in Matrix 4.7.

If Angela is deterred by Bert’s threat and chooses not trespass she loses the respect of other walkers including her friends in the rambling club and has feelings of inadequacy. This is represented by her pay-off of –10. If she decides to trespass on Bert’s land and Bert prosecutes, she is greatly inconvenienced even if she doesn’t end up losing in court. This possibility is represented by her pay-off of –100. If Bert doesn’t prosecute she is personally satisfied and also wins respect from other hikers who are likely to follow her example. This is represented by
her pay-off of 100. Although Angela moves second her choices are simple. She either trespasses or not; her moves correspond to her strategies.

Bert moves again after Angela and his choices are contingent on what Angela does. If Angela trespasses he either prosecutes or not. But if Angela doesn’t trespass then he doesn’t prosecute, he does nothing, and in effect the game ends. He doesn’t really have to make a choice as it would make no sense for him to prosecute if Angela doesn’t trespass. He therefore has two strategy choices: prosecute if Angela trespasses and do nothing if she doesn’t, (prosecute, do nothing), and not prosecute if she trespasses and do nothing otherwise, (not prosecute, do nothing). If Angela doesn’t trespass Bert retains his privacy and his control over access to his land. This satisfactory state of affairs for Bert is represented by his pay-off of 100. If Angela trespasses Bert either attempts to prosecute her or does not. If he prosecutes he is doomed to failure and this is represented by his pay-off of –100. If he decides not to prosecute he just looses face but his threat to prosecute other walkers in the future is considerably weakened. This is represented by his pay-off of –10.

**Matrix 4.7 Strategic form for trespass**

<table>
<thead>
<tr>
<th></th>
<th>trespass</th>
<th>not trespass</th>
</tr>
</thead>
<tbody>
<tr>
<td>prosecute, do nothing</td>
<td>–100, –100</td>
<td>100, –10</td>
</tr>
<tr>
<td>not prosecute, do nothing</td>
<td>–10, 100</td>
<td>100, –10</td>
</tr>
</tbody>
</table>

In Matrix 4.7 the best responses of the players are identified by underlining the corresponding pay-offs. Two Nash equilibria are identified:

- Nash equilibrium (1): {(not prosecute, do nothing), trespass}.
- Nash equilibrium (2): {(prosecute, do nothing), not trespass}.

The first of these is preferred by Angela (she trespasses but Bert doesn’t prosecute) and the second by Bert (Angela doesn’t trespass). The second Nash equilibrium is supported by Bert’s threat to prosecute if Angela trespasses. But is this threat credible? We can answer this question by checking whether either of the two Nash equilibria are subgame perfect. The extensive form of the game is shown in Figure 4.3. Bert moves first by putting up his ‘trespassers will be prosecuted’ sign. Angela then decides whether to trespass or not at A. If she does Bert decides between prosecution or not at B₁. If she doesn’t trespass the game moves to B₂ and Bert does nothing.
To check whether either of the Nash equilibria identified in the strategic form is also subgame perfect we can use backward induction to work back to Bert’s decision node at $B_1$ (we know that at $B_2$ he does nothing). Nash equilibrium (1) is $\{(\text{not prosecute, do nothing}), \text{trespass}\}$. In this equilibrium Bert doesn’t prosecute at $B_1$. This is a rational response for Bert. His pay-off is $-10$ if he doesn’t prosecute but $-100$ if he does.

Nash equilibrium (2) is $\{(\text{prosecute, do nothing}), \text{not trespass}\}$. In this equilibrium Bert prosecutes if Angela trespasses. However, Bert’s threat to prosecute is not tested as Angela is deterred from trespassing. But as we have seen Bert’s pay-offs mean that prosecuting is not a best response for him if Angela does trespass. Therefore the threat to prosecute by Bert is not credible; it is an empty threat. With common knowledge Angela can work this out so she will not be deterred by Bert’s threat. Instead she will trespass on Bert’s land. By trespassing she receives a pay-off of 100 but if she doesn’t trespass her pay-off is $-10$.

Because Bert’s threat to prosecute is not credible only Nash equilibrium (1), $\{(\text{not prosecute, do nothing}), \text{trespass}\}$, is subgame perfect. The path of this equilibrium is indicated by the thickened branches in Figure 4.3.

The analysis of trespass shows how the concept of a subgame perfect Nash equilibrium rules out outcomes supported by empty threats — in this case the threat by Bert to prosecute. There are other empty threat situations that can be modelled as games. For instance, the threat by an employee to resign from his job if he is not given a rise is likely to be an empty one in a recession or when there are no other employers looking for his particular skills in the locality. Similarly the threat by a union to strike may not be credible if the union has only limited strike funds. The threat by a wife to leave her husband (or vice versa) may also be empty if she (or he) has nowhere else to go (see Chapter 9, Section 3). On the international stage the threat to invade a country may not be credible if the decision makers of the invading force are divided.
However, some non-credible threats can be made credible through commitment. You will see this modelled formally in the next section but in trespass Bert may be able to commit to punishing Angela if she trespasses by changing his threat. Instead of threatening to prosecute trespassers he could put a bull in his field. The bull would effectively commit Bert to punishing trespassers and would probably deter Angela. The bull works as a commitment as the bull itself is not worried about the consequences of attacking trespassers so will attack indiscriminately. Contrast this situation with the one where Bert himself ups the ante by threatening to shoot trespassers. The law in a country like the UK is unlikely to make this a best response and therefore it would not be a credible threat.

4.4 Entry deterrence

The structure of the entry deterrence game considered here is very similar to that of trespass. However the question of entry deterrence in relation to market structure and competition policy has been considered in depth in the industrial organisation literature and has wider implications. The two players in the entry deterrence game are an incumbent monopolist and a firm that is a potential entrant into the monopolist’s market. The entrant chooses between entering the market or not. The entrant will enter the market if by doing so it can make positive profits. If the entrant enters the market the monopolist will no longer be in a monopoly position and consequently its profits will be lower. The monopolist tries to deter entry by threatening to fight entry should it occur by engaging in some kind of aggressive market action. For example, the monopolist may threaten to engage in an expensive advertising war or refuse to share the market by maintaining output. The latter action, sometimes called predatory pricing, would mean that prices would fall if the entrant entered and if they fell low enough this could prevent the entrant from making positive profits. Whatever strategy it threatens to adopt it will be costly for the monopolist as well as the entrant. Three questions are raised by this game. First of all, is the threat to fight entry by the monopolist a credible threat? Second, will it deter entry? Lastly, if the threat to fight doesn’t deter entry is there a way for the monopolist to make the threat to fight credible? We will try to answer each of these questions using game theory.

To model this game we can make some simplifying assumptions about the market. Let’s assume that the total market is worth 10 to the monopolist and if the monopolist concedes to the entrant by sharing, each firm’s pay-off is 5. If the monopolist fights entry both make negative profits of −1. If the entrant doesn’t enter its pay-off is zero. The entrant moves first and decides between
entry and staying out of the market. If the entrant enters the monopolist decides between fighting entry and conceding by sharing the market. If the entrant stays out the monopolist does nothing – in effect the game ends. The monopolist has two strategies: concede if the entrant enters and do nothing otherwise (concede, do nothing) and fight if the entrant enters and do nothing otherwise (fight, do nothing). These pay-offs and strategies are shown in the strategic form in Matrix 4.8. In Matrix 4.8 the pay-offs corresponding to the best responses of both players to each other's strategies are underlined.

**Matrix 4.8  Strategic form of the entry deterrence game**

<table>
<thead>
<tr>
<th></th>
<th>Concede, do nothing</th>
<th>Fight, do nothing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Entrant</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Enter</td>
<td>5, 5</td>
<td>-1, -1</td>
</tr>
<tr>
<td>Stay out</td>
<td>0, 10</td>
<td>0, 10</td>
</tr>
</tbody>
</table>

There are two Nash equilibria in this game. They are:

- Nash equilibrium (1): \{enter, (concede, do nothing)\}.
- Nash equilibrium (2): \{stay out, (fight, do nothing)\}.

In Nash equilibrium (1) the monopolist concedes if the entrant enters and the entrant duly enters. In Nash equilibrium (2) the monopolist fights if there is entry and therefore the entrant stays out. Nash equilibrium (1) is preferred by the entrant. The monopolist prefers Nash equilibrium (2) which is sustained by the threat to fight entry. To check whether either of these Nash equilibria are subgame perfect we need to examine the extensive form of the game. This is shown in Figure 4.4. In the game tree the entrant decides whether to enter or not at decision node E. If the entrant enters the monopolist decides between fighting and conceding at M₁.
At M₁ the monopolist’s best response is to concede; by conceding the monopolist’s pay-off is 5 but it is –1 if the monopolist fights. Therefore conceding is a best response by the monopolist to entry. The entrant’s pay-off from not entering the market is zero. Since the monopolist concedes if there is entry the entrant’s pay-off from entry is 5. The entrant will therefore enter at E. This means that only Nash equilibrium (1), {enter, (concede, do nothing)}, incorporates moves that are best responses by the players at all the decision points in the game. In Nash equilibrium (2) the entrant is deterred from entry by the monopolist’s threat to fight at M₁. But fighting at M₁ is not a credible threat and therefore Nash equilibrium (2) is not subgame perfect. Because the threat to fight is not credible only Nash equilibrium (1) is subgame perfect. Figure 4.4.1 highlights the equilibrium path through entry deterrence; at E the entrant enters and at M₁ the monopolist concedes.

4.4.1 Making the threat to fight credible

The theoretical prediction following from the analysis of the game represented in Figure 4.4.1 is that the entrant will always enter and the monopolist will always concede. In the first paragraph of the previous section three questions were posed in relation to this game. The theoretical prediction suggests that the answer to the first two is no: the threat to fight is not credible and entry will not be deterred. But what about the third question? Is there any way to make the threat to fight credible? In this subsection we consider the possibility that the monopolist is able to invest in some commitment to fight which can do just that. Such a commitment could take the form of a non-recoverable or sunk cost that makes fighting optimal but has no benefit for the monopolist otherwise (Dixit, 1980 and 1981). For example, the monopolist could invest in excess capacity. An investment of this sort would only be useful to the monopolist in the event of entry. If the entrant entered the monopolist could increase output at minimal cost. This would lower prices and reduce the profits of the potential entrant. Alternatively, it could invest in goodwill or generating customer loyalty. With a strong customer base the monopolist could confidently start an
advertising war in the event of entry. Making this kind of commitment would alter the monopolist’s pay-offs. We can model this by assuming that the commitment costs $c$ but generates net benefits of $d$ if there is entry and the monopolist fights. The extensive form of the entry deterrence game with these changes is shown in Figure 4.4.2.

Using backward induction to work back to the decision node at $M_1$ we can see that if the entrant enters the monopolist will fight entry if condition (4.1) holds:

$$-1 + d > 5 - c$$

If condition (4.1) holds the pay-off to the monopolist from fighting $(-1 + d)$ is greater than the pay-off from conceding $(5 - c)$. As a result, fighting is a best response for the monopolist. This makes the threat to fight credible since it is in the monopolist’s self-interest to carry out the threat of fighting if entry occurs. In these circumstances the entrant will stay out and the monopolist’s pay-off will be $10 - c$. Depending on $c$ this may be less than the monopolist’s pay-off of 5 from concession in the game without commitment. Since the monopolist concedes if no commitment is made, it is only worthwhile for the monopolist to make the costly commitment if condition (4.2) also holds:

$$10 - c > 5$$

The commitment should only be made if both conditions (4.1) and (4.2) are satisfied. If condition (4.1) is satisfied the commitment deters entry and if condition (4.2) is also satisfied the cost of deterring entry is worth paying. Combining conditions (4.1) and (4.2) leads to condition (4.3):

$$5 > c > 6 - d$$

If condition (4.3) is satisfied then fighting is credible and the commitment that makes it credible is worth investing in. In these circumstances (stay out, (fight, stay out), (do nothing, concede), (fight, stay out)) becomes a Nash equilibrium.
do nothing) is not only a Nash equilibrium but also a subgame perfect Nash equilibrium. This shows that in some circumstances (where condition (4.3) or its equivalent is satisfied) a monopolist may be able to invest in a commitment to fighting that makes the threat to fight credible and thereby deters entry. So the answer to the question ‘can the threat to fight be made credible?’ is a qualified yes. And as we shall see in Chapter 7 if there is uncertainty about the monopolist’s pay-offs the monopolist may not actually need to make the costly commitment in order to deter entry.

The entry deterrence game is another classic game in game theory. In some form or other it invariably appears in textbook introductions to game theory and in economic analyses of imperfect competition in product markets. It will appear without fail in courses in industrial economics or industrial organisation and will very probably make an appearance in courses in managerial and business economics. The game’s defining characteristics are that (i) moves are made sequentially, (ii) one player makes a threat in order to deter some action by a second player, and (iii) the action in question is potentially advantageous to the second player but damaging to the first. Games with these characteristics have many applications outside the theory of industrial organisation. Applications hinge around the question of whether a player’s threat is credible and therefore deters the relevant action of the other. Trespass and the FDI game (where Beta implicitly threatens to export if Alpha undertakes FDI) are both games with this kind of structure. There are other examples that we could examine. For example, a union’s threat to strike in support of a wage demand (see Problem 3 at the end of this chapter) or one person’s threat to sue another could be analysed using the methodology of this section.

In the examples we have looked at in this chapter most of the threats made were not credible; nice–not so nice was an exception in this respect. However, you have seen that players may be able to make their threats credible by investing in some commitment to carry out the threat. For example, in the FDI game Beta could make a commitment to export (possibly by investing in capacity). This could deter Alpha from making the direct foreign investment. In wage negotiations a union could make the threat to strike credible by holding and winning a pre-strike vote. But for a commitment of this kind to be made the potential gains must outweigh the costs.

In this section we look at a family of games commonly called centipede games (because of the way they are represented diagrammatically (see Figure 4.5.2) and some questions will be raised about the backward induction method. For a more detailed discussion of centipede games and the implications of these games for backward induction and subgame perfect Nash equilibrium see the analysis in Kreps (1993: 110–11) on which this section draws or Rosenthal (1981).
Take a look at the extensive form of the baby centipede game in Figure 4.5. In baby centipede there are two players, A and B. A moves first at A₁ and decides between down (D) and right (R). If A chooses down the game ends and both players receive a pay-off of 3. If A chooses right then B chooses between down (d) and right (r). If B chooses down the game ends and A receives a pay-off of 10 and B’s pay-off is 0. If B chooses right then A chooses again between down and right at A₂. If A chooses down A’s pay-off is 1 and B’s pay-off is −10. If A chooses right A’s pay-off is 2 and B’s pay-off is 1.

Working back from the end of the game A’s best option at A₂ is to choose right. Anticipating this B will choose (r). In further anticipation of B’s move A should choose down at A₁. Thus the subgame perfect Nash equilibrium is that A chooses D at the start of the game as A anticipates that B would choose r given the chance and therefore the most A can hope for by choosing right at A₁ is 2. The only reason for A to choose right at A₁ would be if A expected B to choose down but if B expects A to choose right at A₂ then B has no reason to choose down. Therefore if both players are rational and believe the other to be the same the game will end at A₁. Given the pay-offs this subgame perfect Nash equilibrium doesn’t appear unduly problematic.

Now consider the mini-centipede game in Figure 4.5.1. Does this centipede game look familiar? It should do. From A’s decision node at A₂ mini-centipede is the same as baby centipede. Knowing this you should be able to work out that the subgame perfect Nash equilibrium of mini-centipede still has A choosing down, D, at the start. This is because A anticipates that B would choose r at B₁ given the chance, in the further anticipation that A will choose D at A₂ (A’s subgame perfect Nash equilibrium move in baby centipede). In other words A doesn’t expect B to choose d at B₁ which would be a reason for A to choose right instead of down. However, the extra complexity in mini-centipede makes this subgame perfect Nash equilibrium somewhat less intuitive than that of baby centipede. To see this ask yourself what would B think if instead of choosing down at the start of the game A chose right (R)? Would B still be as confident that A would choose down at A₂? Maybe not. And if not could B rely on A choosing right at A₃? If B has any doubts about A’s future moves then B could choose down at B₁ if A chooses right at A₁. If A attaches a high probability¹³ to this possibility then it would be entirely rational for A to choose right at A₁.

Figure 4.5 Baby-centipede
This kind of reasoning weakens the prediction that A will choose down at the start of the game. Now consider a more standard representation of the centipede game in Figure 4.5.2.

Working backwards from the end of the centipede game you should find that once again the subgame perfect Nash equilibrium move for A is to choose D from the start. But this equilibrium is clearly Pareto inefficient. Both players would be better off if the game moved beyond B’s first decision point at B1 with B choosing right. And what should B think if A chooses R at A1? Has A made a mistake or has A deviated with a purpose, and if so what purpose? Can B rely on A choosing R again at A2? If A could be relied on to choose R at A2 this would give B an incentive to choose r at B1 in which case both A and B could benefit by B choosing r at B2. The possibility of A choosing R at A1, even if by mistake, suggests that a completely different outcome for the game is possible, one in which both players are potentially much better off. Consequently, the subgame perfect Nash equilibrium may not be the best prediction of this game. This possibility suggests that there are some limitations of the subgame perfect Nash equilibrium concept and the backward induction method. Experimental evidence tends to support this conclusion. A typical finding is that subjects rarely choose the equivalent of down (take in experiments) straightaway. However, the observed probability of choosing down (or take) does tend to rise as the game progresses, and perhaps surprisingly when the stakes are higher (see, for example, McKelvey and Palfrey, 1992 or Camerer, 2003: 94–5, 218–21 for a summary of this evidence).
In this chapter you have seen how dynamic or sequential-move games are modelled. Five games were analysed in detail: the FDI game, nice–not so nice, trespass, entry deterrence and the centipede game. You saw how there is not always a one-to-one correspondence between a player’s moves and strategies in sequential-move games. If one player moves after another, their moves are contingent on the moves of the other player. The strategies of the player will therefore need to allow for all eventualities. This means that players’ strategies will sometimes have to specify moves at decision nodes that are never actually reached in the equilibrium of the game.

You used backward induction to make a theoretical prediction about the outcome of games with sequential moves. Backward induction allows the analyst to rule out strategies that incorporate non-credible threats. A non-credible threat is a threat that a player would not carry out if called upon to do so as it would not be in the player’s self-interest so to do. Ruling out non-credible threats ensures that strategies specify moves that are a Nash equilibrium in every subgame of the whole game. Only strategies that meet this requirement can be part of a subgame perfect Nash equilibrium.

The role of credibility and commitment in sequential games was further highlighted in the analysis of the entry deterrence game from industrial organisation theory. In the entry deterrence game an incumbent monopolist is threatened by entry. In order to deter entry the monopolist threatens to fight entry should it occur. But if the threat to fight is not credible the entrant will enter. However, the monopolist may be able to make a commitment that makes fighting entry a best response. In these circumstances the threat to fight is credible and entry will be deterred. Because games with the same or a similar structure to entry deterrence are so ubiquitous, you will see this game again when we allow for more of the complexities of life in later chapters. In the last section of this chapter you saw how the subgame perfect Nash equilibrium of the centipede game is Pareto inefficient. This possibility suggests that the subgame perfect Nash equilibrium of a game might not always be the best prediction of the game’s outcome.
4.1

**Matrix 4.5.1** Best responses of Alpha and Beta underlined

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>FDI</td>
<td></td>
</tr>
<tr>
<td>25, -5</td>
<td>40, 10</td>
</tr>
<tr>
<td>export only</td>
<td></td>
</tr>
<tr>
<td>30, 30</td>
<td>60, 10</td>
</tr>
<tr>
<td>export, not export</td>
<td>export, not export</td>
</tr>
<tr>
<td>not export, not export</td>
<td>export, export</td>
</tr>
<tr>
<td>60, 10</td>
<td>30, 30</td>
</tr>
</tbody>
</table>

The Nash equilibria are \{export only, (export, export)\} and \{FDI, (not export, export)\}.

4.2

The subgame perfect Nash equilibrium of nice–not so nice 2 is \{not so nice, (nice, nice)\}, the corresponding branches are highlighted in Figure 4.2.4. Any threat by Two to play not so nice at 2_B in order to persuade One to choose nice is not credible because at 2_B Two’s best response is to choose nice. By choosing nice at 2_B her pay-off is 2 while if she chooses not so nice her pay-off is only 1. In this version of the nice–not so nice game One has an advantage by moving first. One can choose not so nice and then rely on Two being nice which gives One his maximum pay-off of 5.

One’s position in nice–not so nice 2 can be contrasted with his position in nice–not so nice 1. In nice–not so nice 1, even though One still moves first, this does not give him such an advantage because Two’s threat to play not so nice at 2B is credible. One cannot secure his maximum pay-off by choosing not so nice and does better by being nice to Two.
1. The game in Figure 4.6 is played between players A and B. A moves first and chooses between north, east, west or south at A. B moves second and chooses between left and right. How many proper subgames does the sequential move game in Figure 4.6 have and what is the subgame perfect Nash equilibrium of this game?

![Figure 4.6: How many subgames?](image)

2. Consider the following scenario: a single firm monopolises a market. When faced by the possibility of entry into the market the monopolist threatens to fight should entry occur. Use game theory to analyse this scenario and to characterise the circumstances when the threat by the monopolist to fight entry (a) is not credible and (b) is credible.

3. In the wage demands game represented in Figure 4.7 a union is negotiating with a firm and trying to secure a wage increase for its members, the firm’s employees. The union (U) has to decide between making a high or a low wage demand. The firm (F) will definitely accept the low wage demand (at \( F_l \)) but may reject a high wage demand at \( F_h \). If the firm rejects the high demand the union and the employer enter into a long, drawn-out bargaining phase that is expensive for them both (it may for instance involve a work-to-rule, strike or even a lockout). At the end of this phase the
agreed wage will lie somewhere between the original high and low demands; both sides will have made concessions. The pay-offs in Figure 4.7 are illustrative of this scenario. What is the subgame perfect Nash equilibrium of this game? If the firm’s pay-off from rejecting the high demand is 5 instead of 15 does the subgame perfect Nash equilibrium change?

**Questions for discussion**

1. How does the idea of a subgame perfect Nash equilibrium rule out non-credible threats?
2. Explain why all Nash equilibria are not subgame perfect.
3. What is implied by backward induction? Does backward induction always make sense?
4. In games like entry deterrence how can the threat to fight entry or its equivalent be made credible?
5. What is the centipede game and why is the subgame perfect Nash equilibrium of the centipede game somewhat unsatisfactory?

**Answers to problems**

1. The game in Figure 4.6 has 4 proper subgames: the subgames beginning at the decision nodes labelled $B_1$, $B_2$, $B_3$ and $B_4$. The subgame perfect Nash equilibrium is  {south, (left, left, left, left)}. B will always choose left so A should choose south.
2 See Sections 4.4 and 4.4.1. For part (a) you can model a game with the same pattern of relative pay-offs as those in Figure 4.4. For part (b) the relative pay-offs should have the same pattern as those in Figure 4.4.2.

3 In the subgame perfect Nash equilibrium of this game the union makes the low wage demand and the firm accepts it: \{low demand, (reject, accept)\}. The union makes the low wage demand because the firm’s threat to reject is credible; the firm’s equilibrium strategy is (reject, accept). The pay-offs imply that rejection and concession (on both sides) hurts the firm less than passive acceptance of the high demand. If the firm’s pay-off from rejecting the high demand was 5 instead of 15 (implying that the firm’s negotiating costs are higher) the firm’s threat to reject would no longer be credible and in the subgame perfect Nash equilibrium the union would make the high demand and the firm would accept: \{high demand, (accept, accept)\}.

Notes

1 The strategic role of FDI or multinational investment has been considered by a long line of authors probably beginning with Knickerbocker (1973). Smith (1987) and Horstmann and Markusen (1987) develop early models where overseas production is undertaken, in preference to exporting, in order to deter entry as domestic firms in a foreign country become more efficient.

2 Or rollback as Dixit and Skeath (1999) call it.

3 The significance of the qualification will be made clearer in Chapter 5, Section 5.1.

4 Gibbons (1997: 135) defines a subgame as a ‘piece of an original game that remains to be played, beginning at any point at which the complete history of the play of the game thus far, is common knowledge’. See Bierman and Fernandez (1998: Section 6.5) or Montet and Serra (2003: 104) for a more formal definition.

5 Katz and Rosen (1998, Chapter 15: 513) call this the ‘credibility condition’.

6 In FDI the subgames starting from B1 and B2 and ending at terminal nodes are proper subgames. The game starting at A is the whole game and technically the whole game is also a subgame as it starts at a node where there is no uncertainty and ends at a terminal node, but it is only a subgame in a trivial sense.

7 Nice–not so nice is similar to the trust game analysed in Gibbons (1997).

8 The law as it stands in England will heavily punish this kind of action if it is deemed unreasonable force. This was demonstrated when Tony Martin, a householder who shot dead a would-be burglar, received a five year prison sentence (see www.tonymartinsupportgroup.org).

9 See Vickers (1985) for an early introduction.

10 The economic literature relating to entry deterrence generally (see, for example, Bain, 1968, 1956 and Sylos-Labini, 1962) and strategic entry deterrence in particular (see, for example, Spence, 1977) is large.
11 In trespass Bert’s purchase of a bull would constitute a ‘strategic’ investment of this kind.

12 Centipede games have some of the features of the nice–not so nice game as both games require an element of trust for the players to be ‘nice’ to each other. But in centipede games the players face multiple decision nodes.

13 See Rosenthal (1981) for a more rigorous discussion along these lines.